Reliability and probability of failure in structural fire safety engineering

An introduction

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I. Why use probabilistic calculations?
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Perfect safety does not exist
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It’s the basis of the Eurocodes and BS
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Decision making and cost optimization
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Decision making and cost optimization

II. Basic concepts for calculating failure probabilities

III. Discussion
Perfect safety does not exist

Every structure or structural element has a probability of failure
The load effect exhibits random variations

(Baldwin et al., 1970)
Material properties exhibit random variations

Figure II.2: Observed mean minus specified strength and standard deviation of standard cube strength of 88 production units of concrete grade C35 (Rackwitz 1983)

(Caspeele, 2010)
Engineering models and calculation tools are imperfect

(Soetens, 2015)
Thus:
Even for a good design, the load $E$ may be larger than the resistance $R$
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Situations with failure
Limiting the probability of failure by design

The basis of the Eurocodes
The Eurocode specifies a target safety level / failure probability

Target reliability index in function of the consequences of structural failure (normal design conditions)

- Eurocode partial safety factors derived from the target safety level
- Application of Eurocode design rules results in a safety level of 3.8 (generally slightly higher because of conservatism)
Unfortunately, no target is specified for structural fire safety

But options do exist

- Target reliabilities in the European Natural Fire Safety Concept (background documents)

- Back-calculation of the BS target reliability index for specific cases

- Cost-optimization: What is the optimum level of structural fire resistance?

(Van Coile, 2015)
Is there an optimum level of investment in structural fire safety?

Decision making and cost optimization
Is there an optimum level of investment in structural fire safety?

Utility function: \[ Y(p) = B(p) - C(p) - D(p) \]

- Benefit function
- Initial construction cost
- Expected costs due to failure and partial damage

Optimization parameter: \( p \)

Includes:
- [Annual] Probability of a fully developed fire \( (\lambda^*) \)
- **Probability of failure** given a fully developed fire \( (P_{f,fi}) \)
- Failure costs and repair/reconstruction costs in case of partial damage \( (\xi) \)

Optimization criterion: Maximize \( Y \)
Is there an optimum level of investment in structural fire safety?

ISO 834 fire exposure, given $\lambda^*$

Safety investment cost factor

120 min ISO 834 fire exposure
But cost-optimization is no silver bullet

- Parameters of cost-optimization are uncertain
- Stakeholders may not all agree on each input parameter
- Case specific evaluations are not always feasible need for general rules

Determine an Acceptable Range for the structural fire resistance time

Based on results of cost-optimization, i.e.
- failure probabilities
- fire ignition frequencies
- failure costs...
An acceptable range for the structural fire resistance time

Acceptable Range

$\xi$ failure cost ratio

$\xi = 350$

Avoiding overinvestment

Avoiding underinvestment
Basic concepts for calculating failure probabilities
A conceptual introduction
The limit state function $Z$

How do you define failure?

General formula

$$Z = R - E$$

\[\begin{align*}
\text{Failure} & : Z = R - E < 0 \\
\text{Safe/Success} & : Z = R - E \geq 0
\end{align*}\]

Use Monte Carlo methods

$$P_f = I[Z = R - E < 0]$$

Or evaluate/know the PDF

“Load” $E$

“Resistance” $R$

Situations with failure
The limit state function $Z$

Application to fire-exposed structural members: cross-section based

**General formula**

$$Z = R - E$$

**Pure Bending**

$$Z = K_R M_{R,fi,t} - K_E \left( M_G + M_Q \right)$$
The limit state function $Z$

Application to fire-exposed structural members: advanced models (2\textsuperscript{nd} order effects)

General formula

$$Z = R - E$$

But the moment $m_E$ depends on the deflection...

Doomed to computationally expensive Monte Carlo?
A detour to a feasible calculation method...

Even for very complex models, the PDF of a scalar model output $Y$ can be approximated “quickly”

Simple example

$$Y = 2 \frac{X_1 X_2}{X_3}$$

With:

- $X_1 : \text{LN}(3; 0.3)$
- $X_2 : \text{LN}(4; 0.5)$
- $X_3 : \text{LN}(2; 0.2)$

$Y : \text{LN}(12.48; 0.646)$
The limit state function $Z$

Application to fire-exposed structural members: advanced models (2\textsuperscript{nd} order effects)

General formula

$$Z = R - E$$

As there a scalar model output which is representative for the resistance $R$?

For every column realization there is a maximum load $P_{\text{max}}$ (taking into account 2\textsuperscript{nd} order effects) = run model to failure

10000 MCS

PDF approximation from 25 model realizations

Figure 4. ME-MDRM result for CDF describing $P_{\text{max}}$ at 60 minutes ISO834 for $e = 0.05$ m, and comparison with histogram of 10000 MCS, and a lognormal approximation (with parameters based on the MCS).
**The limit state function Z**

Application to fire-exposed structural members: advanced models (2\textsuperscript{nd} order effects)

**General formula**

\[ Z = R - E \]

**Eccentric loading incl. 2\textsuperscript{nd} order**

\[ Z = K_R P_{\text{max,fi,t}} - K_E(P_G + P_Q) \]

Failure probability evaluation feasible with PDF of \( P_{\text{max}} \) approximated
Discussion
How to define “failure” for structural systems exposed to fire?
Need to define a limit state function $Z$

And determine a representative scalar model output $Y$

General formula

$$Z = R - E$$

Options

1. Run the model for ISO 834 exposure till failure $t_R$

$$Z = K_R t_R - K_E t_E$$

2. Run the model for increasing fire load till failure $q_{max}$

$$Z = K_R q_{max} - K_E q_E$$

3. Determine a representative failure indicator e.g $v_{max}$

$$Z = K_R v_{max} - K_E v_{limit}$$

To apply reliability concepts to structural systems exposed to fire, we need a “performance indicator/criterion”
Summary / Conclusions

Perfect safety does not exist. Every structure or structural element has a probability of failure.

Limiting the probability of failure is the very goal of the Eurocodes.

Probabilistic calculations for cost-optimization and decision making.

Standard reliability calculations are based on a limit state function $Z$.

Need to define performance indicators/criteria for structural fire.
Thank you for your attention!