THE BUCKLING OF SLENDER CONCRETE AND CONCRETE-FILLED COLUMNS IN FIRE

Shan-Shan Huang*, Dorothy Hodgkin Scholar, University of Sheffield**,** United Kingdom
Ian Burgess, Professor, University of Sheffield**,** United Kingdom
Zhao-Hui Huang, Lecturer, University of Sheffield**,** United Kingdom
Roger Plank, Professor, University of Sheffield**,** United Kingdom

Keywords: Transient Thermal Strain, Buckling, Slender Concrete Columns, Concrete-Filled Columns, Fire.

ABSTRACT

Pre-compressed concrete has been observed to acquire a large amount of irreversible strain (called Transient Thermal Strain) when it is heated. This effect appears not to occur when heating precedes the application of compressive stress. The objective of the research reported in this paper is to assess how this phenomenon affects the buckling resistance of slender concrete and concrete-filled hollow-section columns in fire. Preliminary analyses pre3sented in the paper lead to the conclusion that TTS does have an adverse effect on the buckling temperatures of uniformly heated slender concrete columns.

1. INTRODUCTION

Concrete-filled steel hollow-section columns have become increasingly popular recently, especially in the construction of tall buildings. They have a much-enhanced load-bearing capacity at ambient temperature compared to conventional reinforced concrete columns and steel hollow-section columns. This enhanced load-bearing capacity mainly results from the 'composite action' between the two materials. The steel tube provides effective confinement to the concrete infill; in turn the concrete infill prevents the steel tube from buckling locally [1-3]. It is a common belief that confining concrete can dramatically increase its compressive strength. However, the effectiveness of confinement to concrete provided by the steel casing in concrete-filled columns is mainly restricted to ambient temperature; its effectiveness at elevated temperatures is questionable. At temperatures below 250ºC, the thermal expansion of steel is higher than that of concrete, in which case not even the contact between the two

* Corresponding author - Department of Civil and Structural Engineering, University of Sheffield, Sheffield, S1 3JD, United Kingdom. Tel: +44 114 2225725/6 E-mail: s.s.huang@shef.ac.uk

materials can be guaranteed. At higher temperatures, although the confinement may be regained due to the reversed relativity between the thermal expansion of concrete and steel (concrete expands more than steel at temperatures beyond 250ºC), the steel tube may have already experienced considerable strength degradation due to heating, and so can not supply as effective a confinement as it does at ambient temperature. Therefore, it may turn out that only the concrete infill remains capable of resisting load in fire conditions. In this case, if the concrete core is not appropriately designed, the column may face disastrous failure when confinement is lost in fire, although high strength is expected by the designer on the basis of the assumed effectiveness of the confinement. Thus, the behaviour of concrete-filled columns in fire depends considerably on the performance of the concrete infill, whose behaviour is therefore worth investigating.

According to the literature on the mechanical properties of concrete at elevated temperatures, concrete experiences a significant increase in compressive strain when it is heated under a constant pre-compression [4-9]. This additional strain is much larger than the sum of the instantaneously stress-induced strain ε_σ and the basic creep ε_{cr}, and is known as Transient Thermal Strain (TTS). It has been noticed that TTS is absent from concrete which is unloaded prior to heating. Its large magnitude makes it the dominant compressive strain component during first heating, and causes significant relaxation to the thermal expansion. TTS is also non-recoverable; it increases with temperature rise and with loading, but never decreases. Moreover, its occurrence is limited to first heating, and it is not observed during cooling or reheating.

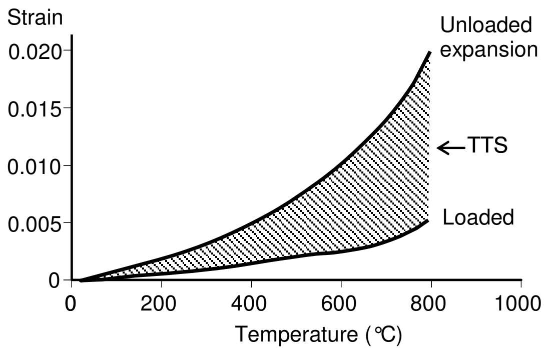

Figure 1: TTS, as the difference between total strain of pre-loaded and unloaded concrete, heated for the first time.

Although TTS is large in magnitude and is not recoverable, it is not usually taken into account in structural analysis. Generally, only its axial effect, relaxing the thermal expansion and shortening the element uniformly across its section, is seen. This is true for stocky elements such as cylinders and short columns, but if the slenderness of a column is high the distribution of transient strain may no longer be uniform when lateral deflection occurs. However, there is no current knowledge about how this large amount of non-uniformly distributed compressive strain may affect buckling. Since concrete-filled columns can be much more slender than RC columns of the same ultimate capacity, due to their enhanced strength, buckling becomes a more significant issue for concrete-filled columns than for the equivalent RC columns.

The objective of this research is to assess how transient thermal strain, in combination with the other temperature-dependent material properties of concrete, affects the buckling resistance of slender concrete and concrete-filled hollow-section columns in fire.

2. INITIAL STUDY OF THE MECHANICS OF INELASTIC BUCKLING

For normal design of slender concrete and concrete-filled columns, the slenderness usually lies in the 'intermediate' range, and the global failure mode is usually inelastic buckling. The simple elastic buckling solution for a perfect strut given by Euler is no longer valid. For the purpose of this research, an insight into the mechanics of inelastic buckling is necessary.

2.1 Inelastic buckling theories

Various inelastic buckling theories have been published since the late 1880s, including the tangent-modulus theory, the reduced-modulus theory and Shanley's theory [10-12]. Both the tangent-modulus theory and the reduced-modulus theory assume that inelastic buckling has characteristics analogous to elastic buckling. It is assumed that a neutral equilibrium state exists, at which the column starts to bend, continuing to failure by buckling at this constant load. These critical loads are determined by generalizing Euler's formula using different Young's moduli, and are known as the tangent-modulus and the reduced-modulus critical loads respectively. The three basic column buckling formulas are written as follows (assuming pinned ends and zero eccentricity):

$$\text{(Euler load)} \qquad F_e = \pi^2 EI/L^2 \qquad (1)$$

$$\text{(Tangent - modulus load)} \qquad F_t = \pi^2 E_t I/L^2 \qquad (2)$$

$$\text{(Reduced - modulus load)} \qquad F_r = \pi^2 E_r I/L^2 \qquad (3)$$

where $F_{(x)}$ = critical load; I = second moment of area of the column cross section; L = column length; E = Young's modulus; E_t = tangent modulus; E_r = reduced modulus.

However, whether a perfect column will remain straight and its equilibrium state suddenly bifurcate at a certain critical load (shown as the straight horizontal lines in Fig. 2) in the inelastic range is questionable. Shanley [10] demonstrated by tests and mathematical analysis that, unlike elastic buckling, inelastic buckling does not have a unique critical load, and that the tangent-modulus and reduced-modulus critical loads are only the upper and lower boundaries of an infinite number of possible buckling loads.

Shanley's theory also indicates that the manner of inelastic buckling is determined by the relative rates of vertical loading and bending in the buckling process. When a column starts to deflect laterally the stresses and strains through its cross-section due to both vertical compression and bending are superposed. If the axial strain takes place more rapidly than the bending strain, then it is possible for buckling to occur without strain reversal, as described in the tangent-modulus theory. On the contrary, if the column deflects so rapidly that only strain reversal on its convex face can keep the load constant, then the process described in the reduced-modulus theory happens. This suggests why the tangent-modulus and reduced-modulus theories give the two practical extremes of inelastic buckling. Between these two extremes, the possible combinations of compression and bending are infinitely variable, explaining why no unique critical load exists for inelastic buckling.

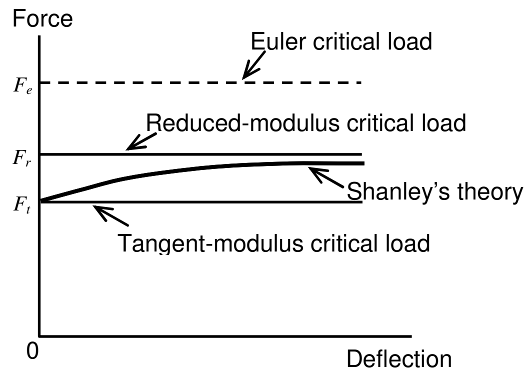

Figure 2: Load-deflection curves given by various column buckling theories.

2.2 Shanley-like column model

Although in practice engineers tend to use the over-conservative tangent-modulus theory to obtain simple and safe solutions to inelastic buckling problems, theoretically only Shanley's theory correctly describes the mechanics of inelastic buckling. Shanley demonstrates his theory with a simplified column model consisting of two rigid legs and an elastic-plastic hinge composed of two axial elements [10]. This model has been modified and extended (as shown in Fig. 3) and its characteristics programmed in Fortran in this study. It has two degrees of freedom:

- Vertical movement u, which is the mean vertical movement of the springs;
- Rotation θ, which is proportional to the differential displacement of the extreme springs.

Since inelastic buckling is significantly rate-related, two dampers, one vertical and one rotational, are added. They respectively control the rates of increase of the two degrees of freedom u and θ. They damp the movement of the model in a controlled manner, which enables the full buckling load-deflection path to be obtained, rather than a sudden bifurcation when the column fails by buckling. In addition, by changing the values of the two damping coefficients the two extreme situations of inelastic buckling (bifurcation at the tangent-modulus or reduced-modulus buckling loads) can be simulated.

The multi-spring model takes into account the material continuity through the cross-section. The axial deformation of each spring is consistent with the usual linear strain-gradient assumption, and hence the mean and differential displacements of each pair of springs at the equivalent locations on the opposite sides are still functions of the two degrees of freedom u and θ. The springs are all identical, having the same stress-strain curves and representing the same material, but the stress level of each spring can differ from the others at any given time, depending on the global deformation and the force equilibrium of the column.

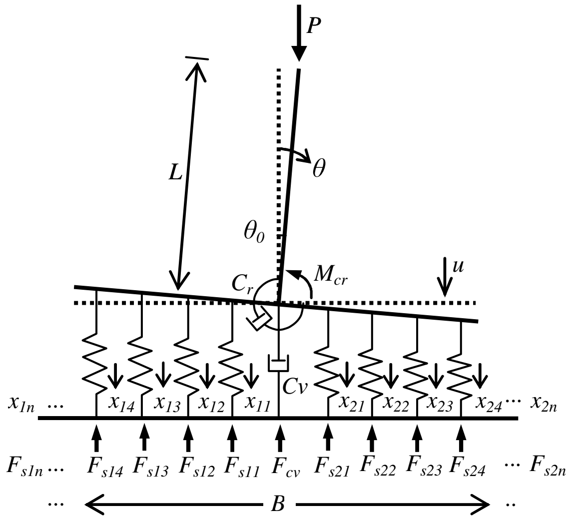

Figure 3: Multi-spring model.

2.3 Dynamic numerical analysis

A dynamic numerical analysis was conducted using a self-coded program based on the multi-spring model. The equations of motion were written for the forces and moments caused by the static imposed load and the dynamic reaction forces on the springs and dampers. Constant loading was imposed on the model. This simulated the application of a weight on top of the column, in a single step but very slowly, so that no initial velocity or acceleration was induced. In the initial time step, the unbalance of the external and internal forces (whose difference is identical to the damping force) induces a velocity, causing the model to move. The model continues to deform gradually through the time steps until a new static equilibrium is reached, and this equilibrium position is recorded. Fig. 4 shows the development of the deformation of each spring through this dynamic procedure, and the corresponding force-deformation relationships are as shown in Fig. 5. The same procedure is repeated for successively higher loads until the rotation of the model is seen to diverge, indicating the final failure by buckling. Plotting all the loads against the corresponding rotations θ, recorded at equilibrium, gives the full equilibrium path, as shown in Figures 6 and 7.

In both Figures 4 and 5, the thick lines represent the springs on the convex side of the bending curvature, while the thin lines represent those on the concave side. Initially, the forces and deformations of the springs all increase almost identically from zero, indicating that very little rotation occurs, and the springs are compressed almost uniformly. At about one second, the deformations and forces of the springs start to differ more markedly from each other, indicating the start of rotation; some of the springs on the convex side experience considerable reverse deformation while the others continue to load. Due to the linear strain-gradient assumption, the differential force and deformation of the pair of springs at the opposite column edges are larger than those of the pair nearest to the centre for a certain global angle of rotation θ. At the end of this load step, the movement of the model gradually stabilizes, as shown in Fig. 4, and the deformation and force of each spring reaches a constant value.

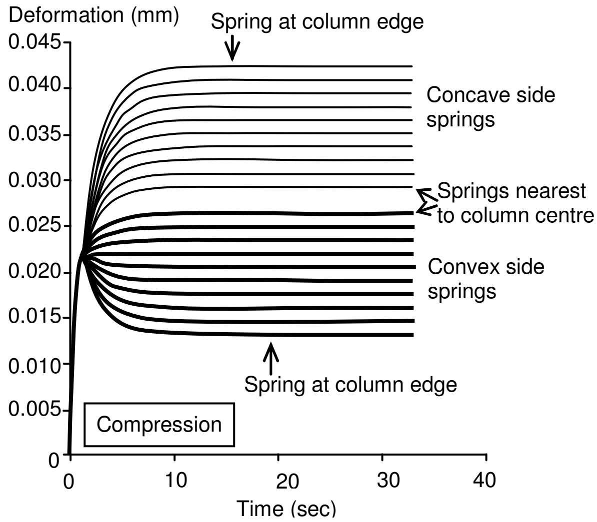

Figure 4: Development of the deformation of each spring over time under constant load as equilibrium is reached.

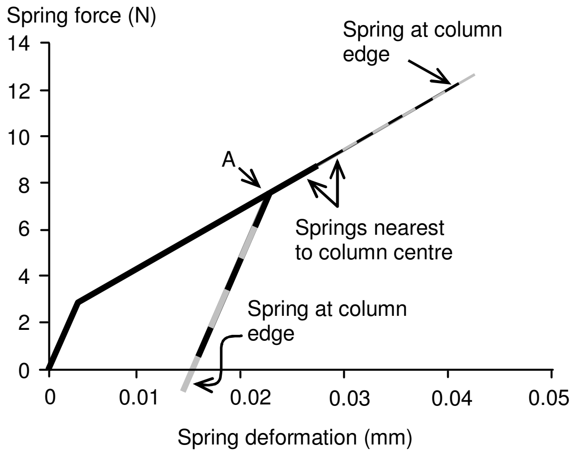

Figure 5: Compressive load-deformation curves of the springs.

The equilibrium load paths of the model with various damping ratios are shown in Fig. 6, with a magnification of the framed part of Fig. 6 being shown in Fig. 7. The analytical results are compared with the theoretical buckling loads: the tangent-modulus buckling load (the short-dashed straight line) and the reduced-modulus buckling load (the long-dashed straight line). Irrespective of the variation of damping, the rotation always starts to increase significantly at the tangent-modulus buckling load and then continues to increase as the force approaches the reduced-modulus buckling load, but the force never goes beyond this upper bound. This shows the column failing by buckling in the exact manner described by Shanley.

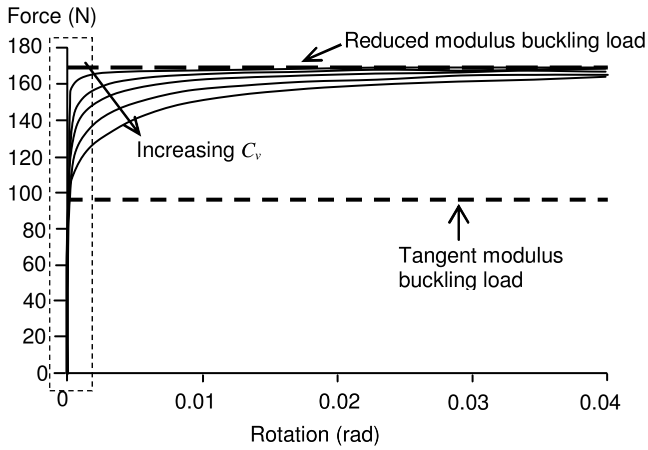

Figure 6: Buckling load paths of the model with various damping ratios.

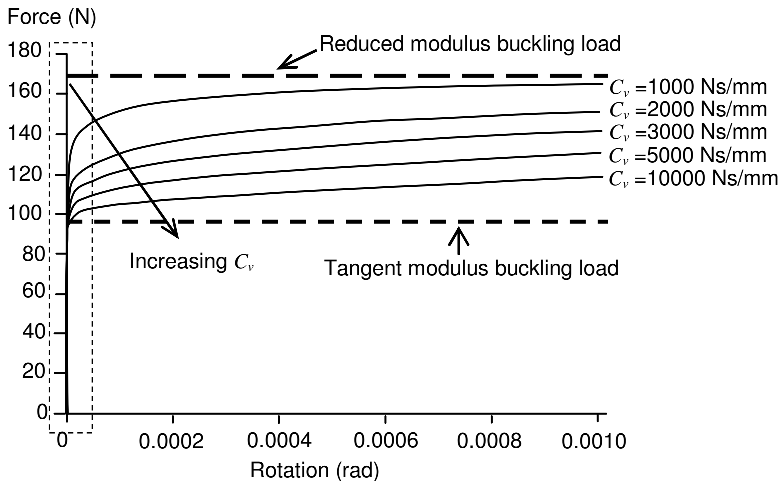

Figure 7: Magnification of the framed section of Fig. 6.

3. ELEVATED-TEMPERATURE ANALYSIS

3.1 Material model of concrete at elevated temperatures

Anderberg's [4] mathematical model of the material properties of concrete at elevated temperatures was adopted in this study. The total strain of uniaxially compressed concrete subjected to elevated temperature consists of four components:

$$\varepsilon = \varepsilon_{th}(T) + \varepsilon_{\sigma}(\sigma) + \varepsilon_{tr}(T,\sigma) + \varepsilon_{cr}(T,\sigma,t) \tag{4}$$

in which ε = total strain; ε_{th} = thermal strain; ε_{σ} = instantaneous stress-induced strain; ε_{cr} = basic creep; ε_{tr} = transient thermal strain; T = temperature; σ = stress, and t = time.

Thermal strain ε_{th} is the strain measured on unloaded concrete subject to a uniform temperature increment. It is a unique function of temperature. Instantaneous stress-induced strain ε_{σ} is the mechanical strain derived from the stress-strain curve. For any given temperature, ε_{σ} is only stress-dependent, but it should be noted that the stress-strain curve may vary with temperature. Transient thermal strain ε_{tr} is found to be reasonably linear with stress. It is also a nonlinear function of temperature and is approximately proportional to ε_{th}:

$$\varepsilon_{tr} = -k_{tr} \frac{\sigma}{\sigma_{u0}} \varepsilon_{th} \tag{5}$$

where k_{tr} = a constant whose value varies from 1.8 to 2.35; and σ_{u0} = the strength at 20°C.

As mentioned by Anderberg, the basic creep ε_{cr} is often small compared to the other strain components, due to the relatively short period of a building fire, and therefore it has been neglected in this analysis.

3.2 Elevated-temperature analysis with Shanley-like column model

After the program describing the behaviour of the Shanley-like model had been evaluated at ambient temperature, it was then upgraded with the high-temperature concrete model described in Section 3.1. At this stage, a uniform temperature distribution across the springs was assumed. The loading procedure was upgraded from the constant loading used in the ambient-temperature analysis to step loading. The load was no longer applied to the column in a single step; it was increased gradually, step by step. Within each load step the load remained constant until equilibrium was reached, and was then increased to the next load value.

Both static-state and transient heating scenarios were applied. In the static-state heating procedure, the step loading was applied at constant temperature until buckling occurred, and the buckling load of the column at this temperature was assessed. By incorporating the transient thermal strain property, pre-loading prior to heating could be taken into account. This loading procedure represents a pre-loaded column which is heated under this load to a stable temperature; it can then be unloaded and reloaded from zero at this temperature until failure occurs. During the transient heating procedure, if the column is loaded by steps to a certain load level, and then temperature was applied as a thermal loading, also step by step until failure occurs, the buckling temperature of the column at this load level can be determined. Obviously, the latter process more directly represents the real loading-heating situation of a column subject to an accidental fire, but it requires an instantaneous change of the material properties from a lower temperature to a higher one when the temperature changes, which causes considerable complexity. These two approaches were compared in order to assess the necessity of introducing this complexity into the research.

The results of the elevated-temperature analysis on an example model, including 20 springs which all have the same temperature, are briefly presented in Fig. 8. This shows the buckling resistances of the model, with and without transient thermal strain, over a range of temperatures, and their comparison with the theoretical buckling loads. The two solid curves show the ultimate buckling loads of the model at various temperatures, with and without considering transient thermal strain (as marked). Their comparison shows that TTS causes a remarkable reduction of the buckling resistance of slender concrete columns under uniform heating. This result is revealing, because the effect of TTS on buckling has hardly been considered in structural analysis, although the uniform-temperature assumption is unrealistic..

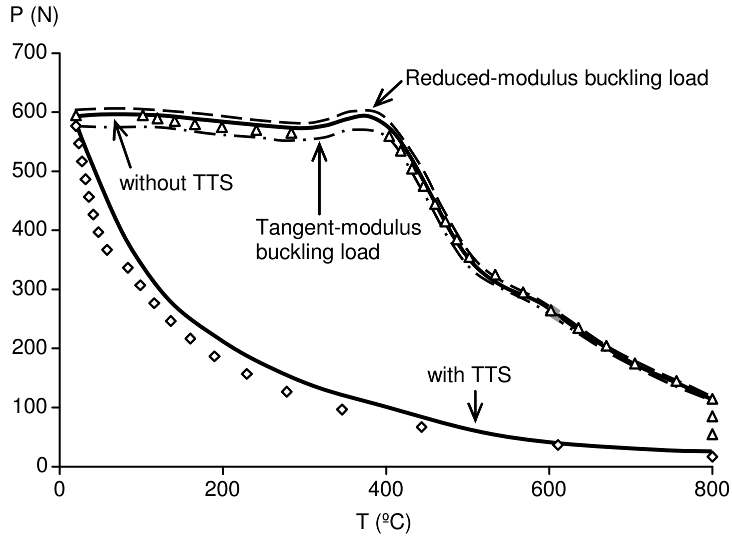

Figure 8: Buckling resistance of the model with and without TTS compared with theoretical buckling loads at elevated temperatures.

On the other hand, this is less surprising if the stress-dependence of TTS is highlighted. The study of the mechanics of inelastic buckling reveals that, when the column starts to bend, the bending causes differential stresses between the concave and convex sides. At elevated temperatures this differential between compressive stresses will cause differential TTS, which further increases the differential total strain between the two sides, leading to further bending. Due to the rather large magnitude of TTS this interactive effect will be significant.

Fig. 8 also shows that the result of the analysis without TTS lies between the two theoretical buckling loads (the two grey dashed curves), which is consistent with Shanley's theory. The results of the transient heating analysis are shown as the black triangles and diamonds plotting the ultimate buckling temperatures of the model at various load levels, with and without considering transient thermal strain. Their comparison with the results of the static-state heating analysis indicates little difference between these two heating scenarios, suggesting that the static-state heating approach is sufficient, at least when the cross-section is uniformly heated, although the influence of the heating scenario is expected to be more significant if a thermal gradient exits through the cross-section. In particular, the magnitude of the additional stresses and strains caused by the thermal gradients should be significantly affected by the stress history.

3.2 Elevated-temperature analysis using *Vulcan*

The analysis on the simplified Shanley-like model was followed by complementary finite element analyses. The FE software *Vulcan*, which has been specially developed for structural fire engineering analysis, was further developed to include transient thermal strain of concrete segments. The 3-noded beam element of *Vulcan*, whose general cross-section consists of a finite number of segments, proved suitable for such development. Each segment is assumed to be fully in contact with adjacent segments, and their relative movements are restricted by the assumption that plane cross-sections remain plane. If the Poisson's ratio of the material is set to zero, these segments should stay reasonably uniaxial, and effectively replace the springs in the Shanley-like model.

As with the Shanley-like-model, the temperature distribution through the cross-section was assumed to be uniform at this stage, and Anderberg's transient thermal strain model was implemented. A three-metre long 200mm wide square bare concrete column, simply-supported and subject to pure compression under transient heating, was examined. The analysis reveals overall buckling as the major failure mode of the column, and the effect of transient thermal strain on this buckling is illustrated in Fig. 9. This figure shows the failure temperatures of the column at various load levels, both with and without considering transient thermal strain. TTS causes reduction of the buckling resistance of this column as it does to the Shanley-like model. The two curves without TTS in Figures 8 and 9 have different shapes, because different stress-strain curves were used in these two analyses; Anderberg's curve for the former and the EC2 curve for the latter.

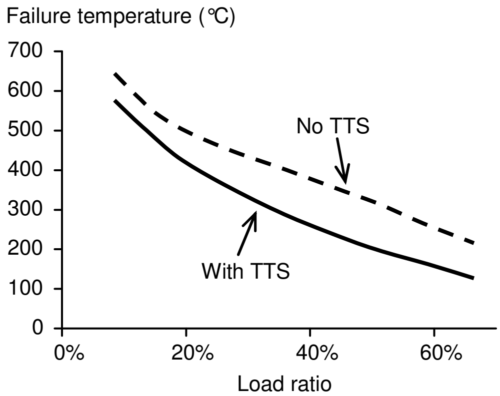

Figure 9: Effect of transient thermal strain on the overall buckling of the column from *Vulcan* analysis.

4. CONCLUSIONS & FUTURE WORK

The classic simplified model due to F.R. Shanley, of a column buckling in the material's inelastic range, has been extended and its characteristics programmed in Fortran in an initial study of the mechanics of inelastic buckling. High-temperature analysis has also been conducted on this model, involving the transient thermal strain property of concrete. This has been followed by complementary FE analyses, developing the software *Vulcan*. Both analyses show a remarkable reduction of the buckling resistance of uniformly heated concrete columns, caused by TTS, revealing the potential dangers of neglecting this property of concrete in structural analysis and design for fire.

In future work, the effects of thermal gradients through the cross-sections of columns due to rapid heating, both symmetric and asymmetric, will be considered. Parametric studies on both concrete and concrete-filled columns, investigating the effects of slenderness ratio, reinforcement and the steel casing will be carried out. The FE analyses using *Vulcan* will then be compared with test results and current design methods. The investigation will later move on to the effect of TTS on columns in continuous structures rather than members in isolation.

Acknowledgment: *The principal author is grateful for the support of Corus Group Ltd and the Engineering and Physical Sciences Research Council of the United Kingdom, under a Dorothy Hodgkin Scholarship.*

6. REFERENCES

[1] Han, L. H., Yang, Y. F., & Xu, L., "An experimental study and calculation on the fire resistance of concrete-filled SHS and RHS columns", *J. Construct. Steel Research*, **59** (4), (2003) pp427-452.

[2] Han, L. H., Zhao, X. L., Yang, Y. F., & Feng, J. B., "Experimental study and calculation of fire resistance of concrete-filled hollow steel columns", *ASCE Journal of Structural Engineering,* **129** (3), (2003b) pp346-356.

[3] Kodur, V. K. R., "Achieving fire resistance in steel columns through concrete filling", *Concrete Engineering International*, **8** (4), (2004) pp50-53.

[4] Anderberg, Y. & Thelanderson, S., *Stress and deformation characteristics of concrete at high temperatures. 2. Experimental investigation and material behaviour model*, Bulletin 54, Lund University, 1976.

[5] Khoury, G. A., *Performance of heated concrete mechanical properties*, Report to the Nuclear Installations Inspectorate of the Health and Safety Executive, London, 1996.

[6] Khoury, G. A., "Strain of heated concrete during two thermal cycles. Part 1: Strain over two cycles, during first heating and at subsequent constant temperature", *Magazine of Concrete Research*, **58** (6), (2006) pp367-385.

[7] Khoury, G. A., Grainger, B. N. & Sullivan, P. J. E., "Transient thermal strain of concrete: literature review, conditions within specimen and behaviour of individual constituents", *Magazine of Concrete Research*, **37** (132), (1985[a]) pp131-144.

[8] Khoury, G. A., Grainger, B. N. & Sullivan, P. J. E., "Strain of concrete during first heating to 600°C under load", *Magazine of Concrete Research*, **37** (133), (1985b) pp95-215.

[9] Schneider, U. & Horvath, J., *Behaviour of ordinary concrete at high temperatures*, Research reports of Vienna University of Technology, Institute of Building Materials, Building Physics and Fire Protection Vol. 9, Vienna, 2003.

[10] Shanley, F. R., "Inelastic column theory", *Journal of the Aeronautical Sciences*, **14** (5), (1947) pp261-268.

[11] Bazant, Z. P. & Cedolin, L., *Stability of structures*, Oxford University Press, Oxford, 1991.

[12] Bleich, F., *Buckling strength of metal structures*, McGraw-Hill, New York, 1952.