USE OF SUB-STRUCTURING IN MODELLING OF COMPOSITE BUILDING RESPONSE TO COMPARTMENT FIRES

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ABSTRACT

In this paper an appropriate procedure, which employs a sub-structuring technique together with selective node re-numbering, for the numerical modelling of building response to fires which are restricted to compartments within the building, is described. In this procedure user-defined cool regions of the building model are sub-structured and condensed into a linear "super-element", which has nodes connected to the non-linear sub-frames located within, and in the immediate vicinity of, the fire zone. The procedure has been incorporated into the non-linear program *Vulcan* which has been developed to model the structural response of loaded structures to fire attack, and two full-scale fire tests have been modelled to examine the computational efficiency of the method. The method makes it much easier than previously to investigate the influence of the surrounding cool structure on the behaviour of elements within the fire compartment, and this has been investigated for these two tests.

KEYWORDS
Super-element, Sub-structuring, Re-numbering, Structural fire behaviour, Finite element analysis.

INTRODUCTION

The performance of structural elements exposed to fire is usually determined by standard fire tests. An important way in which the behaviour of elements in buildings differs from that of similar elements in these furnace tests derives from the very different boundary conditions which are imposed in the two situations. In a real building, structural elements form part of a continuous assembly, and building fires often remain localised, with the fire-affected structure within a fire compartment receiving significant restraint from the cooler areas surrounding it.

Because full-scale fire tests are extremely expensive, the development of analytical methods that can predict the behaviour of building structures when subjected to fire conditions is therefore becoming increasingly important. In recent years a computer program, *Vulcan* (Najjar and Burgess (1996), Huang *et al.* (2003)), has been developed at the University of Sheffield for three-dimensional analysis of the structural behaviour of composite and steel-framed buildings in fire. The program is based on a 3-D non-linear finite element procedure in which composite steel-framed buildings are modelled as an assembly of beam-column, spring, shear connector and slab elements. The analyses are therefore normally very expensive in computing time, especially when modelling large-scale problems.

Fortunately, the design of buildings is influenced by fire safety legislation so that fires are normally contained by internal fireproof compartmentation and remain localised. In the fire condition it is usual to assume that the loading applied generally to the structure is at a level well below that which would cause yield to occur, and it is therefore reasonable to assume that, even during the fire, the adjacent cool structure remains linearly elastic, apart from some areas which are very close to the fire compartment. The objective of the present development was to increase the program's computational efficiency by using the sub-structuring technique together with a re-numbering algorithm.

SUPER-ELEMENT AND RE-NUMBERING PROCEDURE

In this paper a user-defined region of the finite element mesh, which includes only unheated elements, is assumed as a linearly elastic sub-structure (see Figure 1). For this region the relationship between the stiffness matrix $\mathbf{K}_{sb}$, the corresponding displacement vector $\mathbf{U}_{sb}$ and the load vector $\mathbf{R}_{sb}$ is expressed as:

$$\mathbf{K}_{sb}\,\mathbf{U}_{sb} = \mathbf{R}_{sb} \tag{1}$$

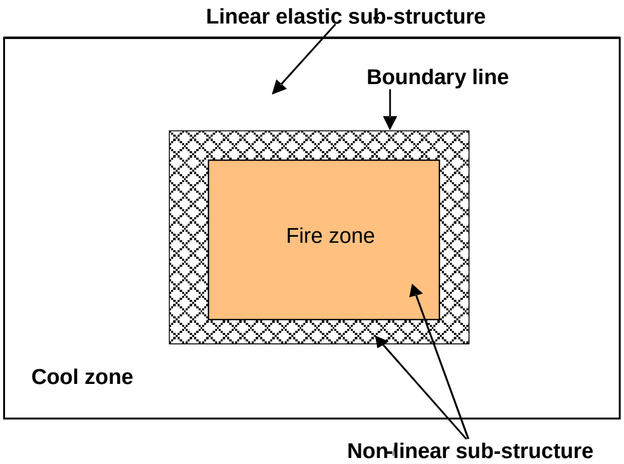

Figure 1: Sub-structuring scheme for structural analysis in fire.

In order to establish a super-element the static condensation procedure is used. Eqn. (1) is partitioned into the form:

$$\begin{bmatrix} \mathbf{K}_{aa} & \mathbf{K}_{ac} \\ \mathbf{K}_{ca} & \mathbf{K}_{cc} \end{bmatrix} \begin{bmatrix} \mathbf{U}_{a} \\ \mathbf{U}_{c} \end{bmatrix} = \begin{bmatrix} \mathbf{R}_{a} \\ \mathbf{R}_{c} \end{bmatrix} \tag{2}$$

where, $\mathbf{U}_a$ is the vector of displacements to be retained (at the boundary nodes between the super-element and the non-linear mesh) and $\mathbf{U}_c$ the vector of displacements to be condensed out (at the other nodes of the super-element), respectively.

The matrix equation for the nodes to be condensed out can be expressed from Eqn. (2) as:

$$\mathbf{U}_c = \mathbf{K}_{cc}^{-1}\left(\mathbf{R}_c - \mathbf{K}_{ca}\mathbf{U}_a\right) \tag{3}$$

The relationship in Eqn. (3) is used to substitute for $\mathbf{U}_c$ into the boundary node equations in Eqn. (2) to obtain the condensed equations

$$\left(\mathbf{K}_{aa} - \mathbf{K}_{ac}\,\mathbf{K}_{cc}^{-1}\,\mathbf{K}_{ca}\right)\mathbf{U}_a = \mathbf{R}_a - \mathbf{K}_{ac}\mathbf{K}_{cc}^{-1}\,\mathbf{R}_c \tag{4}$$

Using

$$\mathbf{K}_{super} = \mathbf{K}_{aa} - \mathbf{K}_{ac}\,\mathbf{K}_{cc}^{-1}\,\mathbf{K}_{ca} \tag{5}$$

and $$\mathbf{R}_{\mathbf{super}} = \mathbf{R}_{\mathbf{a}} - \mathbf{K}_{\mathbf{ac}} \mathbf{K}_{\mathbf{cc}}^{-1} \mathbf{R}_{\mathbf{c}} \tag{6}$$

Eqn. (4) becomes $$\mathbf{K}_{\mathbf{super}} \mathbf{U}_{\mathbf{a}} = \mathbf{R}_{\mathbf{super}} \tag{7}$$

where $\mathbf{K}_{\mathbf{super}}$ and $\mathbf{R}_{\mathbf{super}}$ are respectively the stiffness matrix and load vector of the super-element. The frontal solution method has been used during the condensation procedure for the super-element, to enable the program to model large-scale structures without difficulty. An undesirable effect of the condensation process is that it creates an element with a very large number of nodes, whose stiffness matrix is very highly populated and therefore has a bandwidth very close to the maximum possible for its final degrees of freedom. The node numbering of the remaining sub-structure for non-linear analysis may become very badly conditioned when it is re-assembled with the super-element. This can cause a very much broader bandwidth of the structural stiffness matrix than would occur if the problem were solved without sub-structuring. In general solution time is related to the square of the bandwidth, while it is only linearly related to the dimension of the stiffness matrix. Thus the considerable decrease in the number of nodes considered due to condensation may be offset by the increase in bandwidth, and may in consequence produce no improvement in processing times. It is therefore necessary to re-number the remaining sub-structure including the super-element nodes in order to reduce the bandwidth to a more optimal level. A re-numbering algorithm proposed by Burgess and Lai (1986) has been adopted in this paper. After re-numbering the mapping array is updated. The structural information related to the old numbering system is re-sequenced in the new numbering system and is then ready to start the non-linear analysis. When the stiffness matrix for the super-element has been constructed it is used as a boundary element connected to the sub-structure, which is modelled using normal meshing of non-linear elements. During the subsequent non-linear analysis the stiffness matrix of the super-element $\mathbf{K}_{\mathbf{super}}$ remains constant. The total number of nodes and elements for which non-linear analysis has to be carried out is reduced to those within the selected region, whilst retaining the effect of either the whole or a large part of the structure which is not directly involved in the fire.

NUMERICAL EXAMPLES

The procedures reported above for creating a super-element and re-numbering the residual sub-structure have now been incorporated within *Vulcan*. The program is now capable of being run in three ways: (1) using the original procedure without any sub-structuring or re-numbering; (2) using the original procedure after re-numbering; (3) using the new procedure with a super-element and subsequent re-numbering. Two of the six full-scale compartment fire tests on the composite building at Cardington are modelled here to demonstrate the key effects of the new procedures. The studies were run on a Pentium 4, 1.6GHz personal computer with 1.0Gb of RAM.

The Restrained Beam Test: This test (Bentley et al (1995)) was the first and smallest of the fire tests carried out on the test building. It involved heating a single secondary beam and an area of the surrounding slab on the seventh floor. In this example four cases, all of which are illustrated in Figure 2, have been analysed. In the selection of super-element configurations here only the intuitively sensible arrangements shown in Figure 2 have been considered. The run-times for Case 1 (normal modelling), Case 2, Case 3 and Case 4 are 280, 25.3, 18.8 and 4.3 minutes respectively. The ten-fold time saving between Cases 1 and 2 is at least in part due to the re-numbering process alone; the presence of columns above and below the floor structure is capable of causing very large and sparsely populated bandwidths, although the effect on the skyline is less marked. It is necessary however to check that accuracy is maintained in predicting overall response, and also that the localised behaviour in the zone of interest is not changed radically by connection to the super-element.

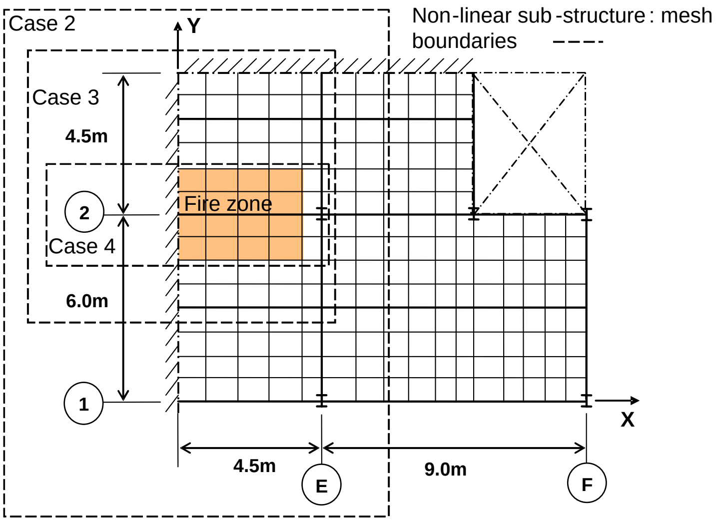

Figure 2 Finite element layout adopted in the analysis for the Restrained Beam Test, together with the boundaries of the non-linear sub-structures for the three cases studied.

In order to examine the effect on overall accuracy of the modelling the mid-span deflections of the heated beam are plotted in Figure 3 for all cases against the temperature of the bottom flange of the beam, together with the measured test results.

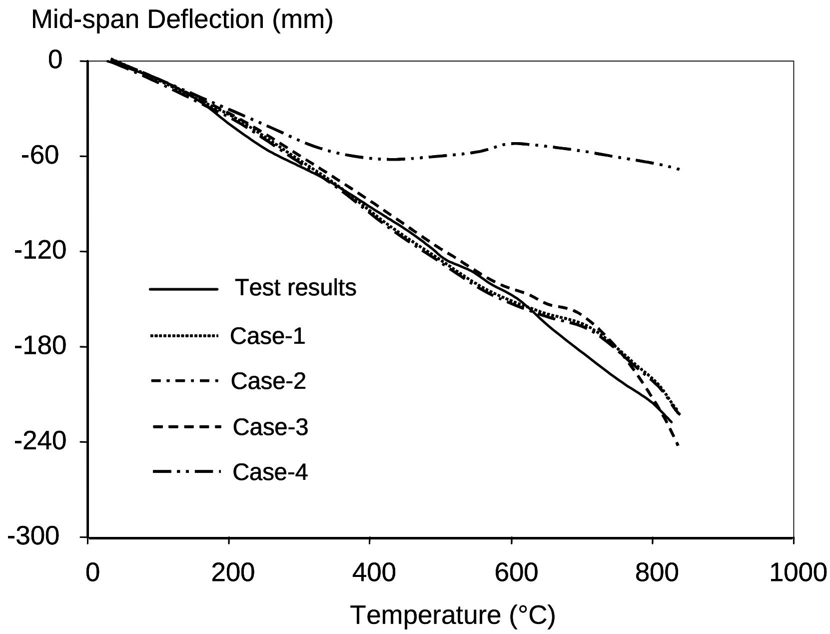

Figure 3 Comparison of predictions with measured min-span deflections for the different non-linear sub-structures analysed in the Restrained Beam Test

It can be seen that Case 2 produces results almost identical to those from normal modelling (Case 1), which itself compares well with the test results. Case 3 involves some minor sacrifice of accuracy in the final range of test temperatures; it may be anticipated that this would increase if the temperatures were increased further. Case 4, in which only the tested beam and the concrete slab within the furnace width are contained within the non-linear sub-frame, shows the effect of eliminating failure in the immediately adjacent zones from the analysis. Below a beam lower flange temperature of 200°C Case 4 deflections are almost identical to those from the other modelling, whereas above 400°C, when the steel loses strength rapidly, the deflections cease as the heated zone effectively re-distributes its load to the surrounding elastic material. The influence of the re-numbering process, taken in isolation from the creation of a super-element, can be gauged from a re-run of Case 1 (normal modelling) after re-numbering. The run-time in this case was 201 minutes, compared with 280, a significant but unspectacular saving. Use of the super-element had the effect not only of reducing the total elements and nodes needed for the analysis, but also of making the non-linear procedure more stable and reducing the average number of iterations needed for each step.

British Steel Corner Test: In July 1995 Test 3 of the British Steel series was carried out (Bentley et al (1996)) in a corner bay of the structure, of dimensions 9.98m wide by 7.57m deep. The test location and the extent of the structure incorporated within the numerical model is shown in Figure 4.

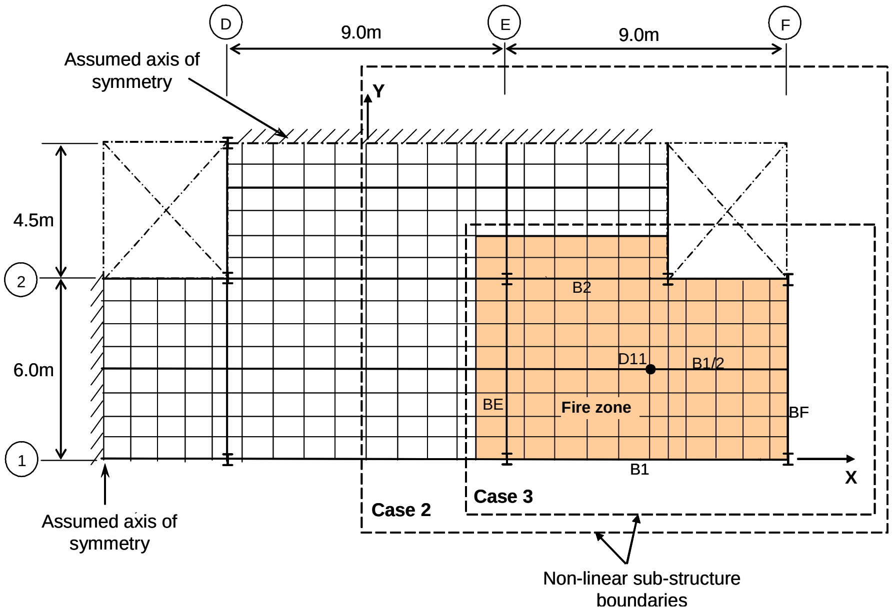

Figure 4 Finite element layout adopted in the analysis for British Steel Corner Test, together with the locations of non-linear sub-structures for the two cases studied

In this example three cases, illustrated in Figure 4, have been analysed. The deflections at position D11, at mid-span of heated beam B1/2, are plotted in Figure 5 against the temperature of the bottom flange of this beam for all three cases, together with the test results. It can be seen that the three cases produced almost identical results, the only small difference appearing for temperatures above 800°C.

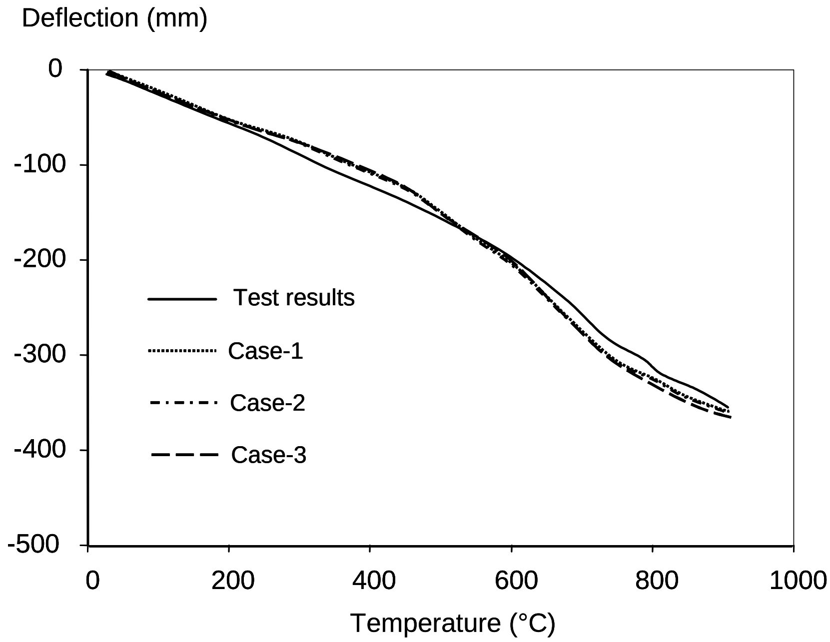

Figure 5 Comparison of predictions with measured deflections at position D11 for different non-linear sub-structures analysed for the British Steel Corner Test

The run-times for Case 1 (normal modelling), Case 2 and Case 3 were 1800, 236, and 212 minutes, respectively. There was a significant run-time saving for Case 2, at 13.1% of Case 1. For Case 3 very little further saving was achieved, at only 11.7% of Case 1, but with a sacrifice of some accuracy in the analysis. Case 1, was also re-run after simply re-numbering the mesh, and the run-time in this case was 851 minutes, or 47% of that for the original Case 1.

Comparing Case 3 for the corner bay test with Case 4 of the Restrained Beam test, both represent extreme situations in which all elements outside the fire compartment are included within the elastic super-element. In the BS Corner Test the fire zone was much more extensive than the Restrained Beam Test, so there was less opportunity for loads in the fire zone to be redistributed to the surrounding cool structure. Additionally, there was little in-plane restraint to thermal expansion of components within the fire compartment, whereas the Restrained Beam Test was completely surrounded by cool slab, providing considerable in-plane restraint. It is therefore logical that the Corner Test results should be less affected by the modelling of the cool adjacent structure than the Restrained Beam Test, so that Case 3 shows almost the same accuracy as Case 1. This contrasts markedly with the Restrained Beam Test, in which thermal expansion is resisted and there is an obvious mechanism available for redistribution of loading.

CONCLUSIONS

This paper illustrates the use of the sub-structuring technique and a re-numbering algorithm to increase the efficiency of modelling the structural behaviour of buildings in fire. The procedure has been incorporated into *Vulcan* and two full-scale fire tests have been modelled, both to show the computational efficiency of the method and to investigate the principles which control selection of super-elements. From the modelling of these fire tests it has been seen that if the surrounding cool structure can provide high restraint to thermal expansion of the fire compartment (as in the Restrained Beam Test) this is a very important influence on the behaviour of the heated structure. Of course, the structural continuity provided by the adjacent structure is also important. For fire compartments subjected to very little in-plane restraint (as in the British Steel Corner Test) structural continuity is the only major factor to influence the behaviour in the fire compartment. It is evident that using this sub-structuring technique to represent the cool structure adjacent to a fire compartment is one of the building blocks in increasing the efficiency of modelling of structures in fire. The procedure proposed in this paper is a combination of sub-structuring and re-numbering, and should therefore maximise computational efficiency as much as possible for any given hardware and solution scheme.

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