3D MODELLING OF BEAM-COLUMNS WITH GENERAL CROSS-SECTIONS IN FIRE

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ABSTRACT

A non-linear finite element, developed for three-dimensional modelling of beam-column elements of general cross-sections in fire conditions, is described. The procedure is based on general formulations proposed by Bathe, with further development to make it suitable for the analysis of steel, reinforced concrete and composite framed structures under the influence of fire. Because of the changes in material properties and the large deflections experienced in fire, both geometric and material non-linearities are taken into account in this formulation. The cross-section of the beam-column is divided into a matrix of segments, and each segment may have different material, temperature and mechanical properties. The more complicated aspects of structural behaviour in fire conditions, such as thermal expansion, cracking or crushing of concrete, and progressive change of the constitutive properties of materials with temperature, are modelled. Since it is possible to offset their nodes by pre-determined distances the elements can easily be combined with shell or plate elements to model composite floor systems in fire. One high-deflection numerical example using linear elastic material is presented to demonstrate the accuracy at high deflections of the elements. A full-scale standard fire test on a composite slim-floor beam has been modelled to show the capabilities of the element. The influences of catenary action on the behaviour of the tested beam are also investigated using different support conditions.

KEYWORDS: *geometrical non-linearity; beam-column element; segmentation; structural fire behaviour; composite structures*

INTRODUCTION

The behaviour of structures exposed to fire is usually described in terms of the concept of fire resistance, which is the period of time under exposure to a standard fire time-temperature curve at which some form of limiting behaviour occurs. Current design codes for steel framed and composite structures [1, 2] allow designers to treat fire as one of the basic design limit states, taking account of:

- Non-uniform heating due to partial protection, which may be inherent in the framing system or specially applied,
- The level of loading at the fire limit state, using partial safety factors lower than those used for ultimate limit states, because of the relative improbability of such accidental conditions,
- Realistic stress-strain characteristics of steel at elevated temperatures.

These approaches can greatly reduce the amount of fire protection required, compared with traditional prescriptive approaches, and in some cases show that no applied protection is needed. Their main limitation is that they are based on the behaviour under test of isolated simply supported members, usually heated according to the standard ISO834 time-temperature curve [3]. In real buildings structural elements form part of a continuous assembly, and building fires often remain localised, with the fire-affected region of the structure receiving significant restraint from cooler areas surrounding it. The real behaviour of these structural elements can therefore be very different from that indicated by standard furnace tests.

In 1995-96 six large fire tests were carried out on a full-scale composite building at the BRE Fire Research Laboratory at Cardington [4]. The tests made it clear that unprotected steel members could have significantly greater fire resistance within real multi-storey buildings than when tested as isolated members. This was undoubtedly due to interaction between the heated members within the fire compartment, the concrete floor slabs and the adjacent steel frame structure. If such interactions are to be used to advantage by designers in specifying fire protection strategies as part of an integrated limit state design process, then this can not practically be based on testing because of the extremely high costs which are implied. It is therefore becoming increasingly important that software models be developed to enable the behaviour of such structures under fire conditions to be predicted with sufficient accuracy.

The specialised finite element program *Vulcan* has been progressively developed over the past decade [5-8] at the University of Sheffield for three-dimensional modelling of the structural behaviour of composite and steel-framed buildings in fire. In this program steel beam-columns have previously been represented by two-noded one-dimensional line elements in which each node has 8 degrees of freedom in local coordinates, which transform into 11 degrees of freedom in global coordinates [5]. Apart from the usual three translational and three rotational degrees of freedom, five extra degrees of freedom are present, representing three derivatives of displacement ("strains"), plus twisting and warping degrees of freedom. Although these elements have proved extremely accurate up to and beyond the deflection levels which are practically acceptable in building structures in fire conditions, there are some major disadvantages to using this approach:

- Due to the presence of the five unconventional degrees of freedom at each node the processing time is increased considerably compared to analysis using the normal six degrees of freedom;
- It is necessary always to apply constraints to some of the nodal degrees of freedom, even when no external boundary conditions exist;

- There are some logical difficulties in setting nodal constraint conditions for sloping members. Different specifications of these constraints for such members can influence the results of the analyses quite significantly, especially when displacements are large [9].

The main objective of this work was to develop a more robust non-linear element for three-dimensional modelling of general beam-column elements of structures in fire conditions.

NON-LINEAR PROCEDURE

Using the general continuum mechanics equations for large-displacement non-linear analysis, the calculation of the non-linear beam element matrices represents a direct extension of the linear (small displacement) formulation. The calculations are performed as in the evaluation of the matrices of the elements with the conventional six displacement degrees of freedom only. The formulation of the element matrices for large-displacement, large-rotation behaviour of a beam with rectangular cross-sectional area is considered first. Using segmentation of the element cross-section as shown in Fig. 1, and allowing some segments to be "void" (with zero mechanical strength and stiffness), different shapes of cross-section can be modelled easily.

The reference axis for a beam-column element, on which the nodes are located, can be placed inside or outside the cross-section of the element. Each node of the element has three translational and three rotational degrees of freedom, in both local and global coordinates. The main assumptions of the model can be summarised as follows:

- Plane sections originally normal to the reference axis remain plane and undistorted under deformation, but are not necessarily normal to this axis. It is assumed that there is no slip between segments.

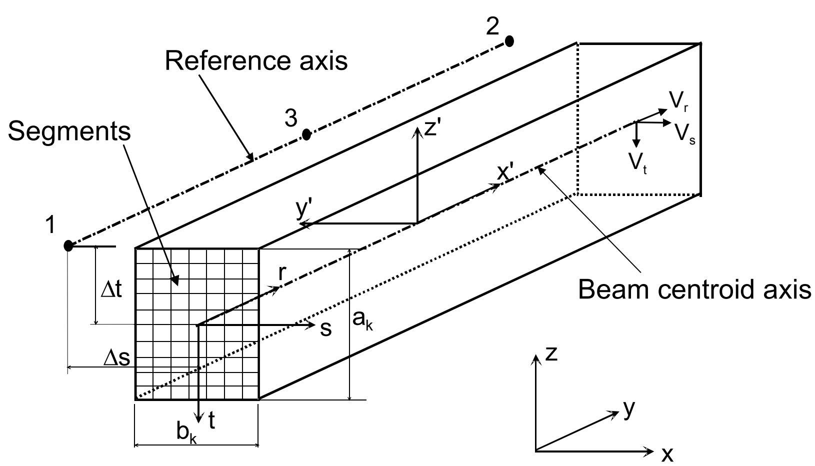

FIGURE 1: Three-dimensional three-noded beam element configuration.

- The displacements and rotations of the element can be arbitrarily large but the element strains are still assumed to be small, which means that the cross-sectional area does not change. This is an appropriate assumption for most geometrically non-linear analyses of beam-type structures [10].

- Each segment within the cross-section can have a different temperature, but this is uniform along the element. The initial material properties of each segment may be different, and the stress-strain relationships may change independently for each segment.
- For each segment only the longitudinal stress and two shear stresses are non-zero.

Using the natural coordinates *r, s, t*, shown in Fig. 1, the Cartesian coordinates of a point in an element with *N* node points at time t are given (see Fig. 1) by

$$\begin{aligned}
{}^t x(r,s,t) &= \sum_{k=1}^{N} h_k\,{}^t x^k + \sum_{k=1}^{N}\left(\frac{t\,a_k}{2}+\Delta t^k\right)h_k\,{}^tV_{tx}^k + \sum_{k=1}^{N}\left(\frac{s\,b_k}{2}+\Delta s^k\right)h_k\,{}^tV_{sx}^k \\
{}^t y(r,s,t) &= \sum_{k=1}^{N} h_k\,{}^t y^k + \sum_{k=1}^{N}\left(\frac{t\,a_k}{2}+\Delta t^k\right)h_k\,{}^tV_{ty}^k + \sum_{k=1}^{N}\left(\frac{s\,b_k}{2}+\Delta s^k\right)h_k\,{}^tV_{sy}^k \qquad (1)\\
{}^t z(r,s,t) &= \sum_{k=1}^{N} h_k\,{}^t z^k + \sum_{k=1}^{N}\left(\frac{t\,a_k}{2}+\Delta t^k\right)h_k\,{}^tV_{tz}^k + \sum_{k=1}^{N}\left(\frac{s\,b_k}{2}+\Delta s^k\right)h_k\,{}^tV_{sz}^k
\end{aligned}$$

in which

$h_k(r)$	= the interpolation functions,
x, y, z	= global Cartesian coordinates,
${}^t x,\ {}^t y,\ {}^t z$	= global Cartesian coordinates of any point in the element at time *t*,
${}^t x^k,\ {}^t y^k,\ {}^t z^k$	= global Cartesian coordinates of nodal point *k* at time *t*,
a_k, b_k	= cross-sectional dimensions of the beam at nodal point *k*,
$\Delta t^k, \Delta s^k$	= offsets of reference axis from the central of the cross-section at nodal point *k* in *t* and *s* directions,
${}^tV_{tx}^k,\ {}^tV_{ty}^k,\ {}^tV_{tz}^k$	= components of unit vector ${}^t\mathbf{V}_t^k$ in direction *t* at nodal point *k*,
${}^tV_{sx}^k,\ {}^tV_{sy}^k,\ {}^tV_{sz}^k$	= components of unit vector ${}^t\mathbf{V}_s^k$ in direction *s* at nodal point *k*,

Using Equation (1) the expressions for the total displacements (u, v, w) and their incremental components ($\Delta u, \Delta v, \Delta w$), in terms of the nodal point values and changes in the direction cosines of the nodal point vectors, can be written as:

$$\begin{aligned}
{}^t u(r,s,t) &= \sum_{k=1}^{N} h_k\,{}^t u^k + \sum_{k=1}^{N}\left(\frac{t\,a_k}{2}+\Delta t^k\right)h_k\left({}^tV_{tx}^k - {}^0V_{tx}^k\right) + \sum_{k=1}^{N}\left(\frac{s\,b_k}{2}+\Delta s^k\right)h_k\left({}^tV_{sx}^k - {}^0V_{sx}^k\right) \\
{}^t v(r,s,t) &= \sum_{k=1}^{N} h_k\,{}^t v^k + \sum_{k=1}^{N}\left(\frac{t\,a_k}{2}+\Delta t^k\right)h_k\left({}^tV_{ty}^k - {}^0V_{ty}^k\right) + \sum_{k=1}^{N}\left(\frac{s\,b_k}{2}+\Delta s^k\right)h_k\left({}^tV_{sy}^k - {}^0V_{sy}^k\right) \qquad (2)\\
{}^t w(r,s,t) &= \sum_{k=1}^{N} h_k\,{}^t w^k + \sum_{k=1}^{N}\left(\frac{t\,a_k}{2}+\Delta t^k\right)h_k\left({}^tV_{tz}^k - {}^0V_{tz}^k\right) + \sum_{k=1}^{N}\left(\frac{s\,b_k}{2}+\Delta s^k\right)h_k\left({}^tV_{sz}^k - {}^0V_{sz}^k\right)
\end{aligned}$$

and

$$\begin{aligned}
\Delta u(r,s,t) &= \sum_{k=1}^{N} h_k\,\Delta u^k + \sum_{k=1}^{N}\left(\frac{t\,a_k}{2}+\Delta t^k\right)h_k V_{tx}^k + \sum_{k=1}^{N}\left(\frac{s\,b_k}{2}+\Delta s^k\right)h_k V_{sx}^k \\
\Delta v(r,s,t) &= \sum_{k=1}^{N} h_k\,\Delta v^k + \sum_{k=1}^{N}\left(\frac{t\,a_k}{2}+\Delta t^k\right)h_k V_{ty}^k + \sum_{k=1}^{N}\left(\frac{s\,b_k}{2}+\Delta s^k\right)h_k V_{sy}^k \qquad (3)\\
\Delta w(r,s,t) &= \sum_{k=1}^{N} h_k\,\Delta w^k + \sum_{k=1}^{N}\left(\frac{t\,a_k}{2}+\Delta t^k\right)h_k V_{tz}^k + \sum_{k=1}^{N}\left(\frac{s\,b_k}{2}+\Delta s^k\right)h_k V_{sz}^k
\end{aligned}$$

in which

$$\begin{cases} V_{tx}^k = {}^{t+\Delta t}V_{tx}^k - {}^tV_{tx}^k \\ V_{ty}^k = {}^{t+\Delta t}V_{ty}^k - {}^tV_{ty}^k \\ V_{tz}^k = {}^{t+\Delta t}V_{tz}^k - {}^tV_{tz}^k \end{cases} \quad \begin{cases} V_{sx}^k = {}^{t+\Delta t}V_{sx}^k - {}^tV_{sx}^k \\ V_{sy}^k = {}^{t+\Delta t}V_{sy}^k - {}^tV_{sy}^k \\ V_{sz}^k = {}^{t+\Delta t}V_{sz}^k - {}^tV_{sz}^k \end{cases} \tag{4}$$

The relationship given in Equation (2) is directly employed to evaluate the total displacements and total strains (hence also the total stresses) and holds for any magnitude of the displacement components. The relationship of Equation (3) is used in the linearisation of the Principle of Virtual Work, and needs to express the components of the changes in the direction cosines $\boldsymbol{V}_t^k$ and $\boldsymbol{V}_s^k$ in terms of the nodal rotation degrees of freedom. Depending on the size of the incremental step, the actual rotations corresponding to the vectors $\boldsymbol{V}_t^k$, $\boldsymbol{V}_s^k$ may be large, and therefore cannot be represented by vector component rotations about the Cartesian axes. However, the objective here is to express the continuum linear and non-linear strain increments by finite element degrees of freedom and corresponding interpolations, so as to achieve a full linearisation of the Principle of Virtual Work. The vector of nodal rotational degrees of freedom $\boldsymbol{\theta}_k$ can be defined using the components measured about the Cartesian axes, sing the second-order approximations [11],

$$\begin{cases} \boldsymbol{V}_t^k = \boldsymbol{\theta}_k \times {}^t\boldsymbol{V}_t^k + \frac{1}{2}\boldsymbol{\theta}_k \times \left(\boldsymbol{\theta}_k \times {}^t\boldsymbol{V}_t^k\right) \\ \boldsymbol{V}_s^k = \boldsymbol{\theta}_k \times {}^t\boldsymbol{V}_s^k + \frac{1}{2}\boldsymbol{\theta}_k \times \left(\boldsymbol{\theta}_k \times {}^t\boldsymbol{V}_s^k\right) \end{cases} \tag{5}$$

where,

$$\boldsymbol{\theta}_k \times {}^t\boldsymbol{V}_t^k = \begin{Bmatrix} \theta_y^k \; {}^tV_{tz}^k - \theta_z^k \; {}^tV_{ty}^k \\ \theta_z^k \; {}^tV_{tx}^k - \theta_x^k \; {}^tV_{tz}^k \\ \theta_x^k \; {}^tV_{ty}^k - \theta_y^k \; {}^tV_{tx}^k \end{Bmatrix} \tag{6}$$

and

$$\boldsymbol{\theta}_k \times {}^t\mathbf{V}_s^k = \begin{Bmatrix} \theta_y^k \; {}^tV_{sz}^k - \theta_z^k \; {}^tV_{sy}^k \\ \theta_z^k \; {}^tV_{sx}^k - \theta_x^k \; {}^tV_{sz}^k \\ \theta_x^k \; {}^tV_{sy}^k - \theta_y^k \; {}^tV_{sx}^k \end{Bmatrix} \tag{7}$$

The only purpose of using $\boldsymbol{\theta}_k$ is to evaluate (approximations to) the new direction vectors; $\boldsymbol{\theta}_k$ is discarded thereafter.

Equations (1) to (5) are the basic interpolations and expressions which are used to establish the strain-displacement interpolation matrices for geometrically non-linear analysis. In this study a Total Lagrangian (TL) formulation is adopted. The details of constructing for element stiffness matrix and internal force vector can be found in Reference [12].

CONSTITUTIVE MODELLING AND PROPERTIES OF MATERIALS

The stress-strain relationships of concrete and steel at elevated temperatures specified by Eurocode 4 [13] are adopted for this study (see Fig. 2). The uniaxial tensile and compressive strengths of concrete are assumed to be related by $f_t^{'}=0.3321\sqrt{f_c^{'}}$ MPa [14]. Hence the concrete tensile strength $f_t^{'}$ changes with temperature. It is assumed that concrete exhibits linear elastic behaviour up to its ultimate tensile capacity. Beyond this point the concrete cracks and the tensile stress decreases gradually with increasing tensile strain, rather than dropping to zero abruptly as would occur in a perfectly brittle material. The constitutive matrix $\boldsymbol{D}_c^{'}$ of a cracked concrete segment becomes

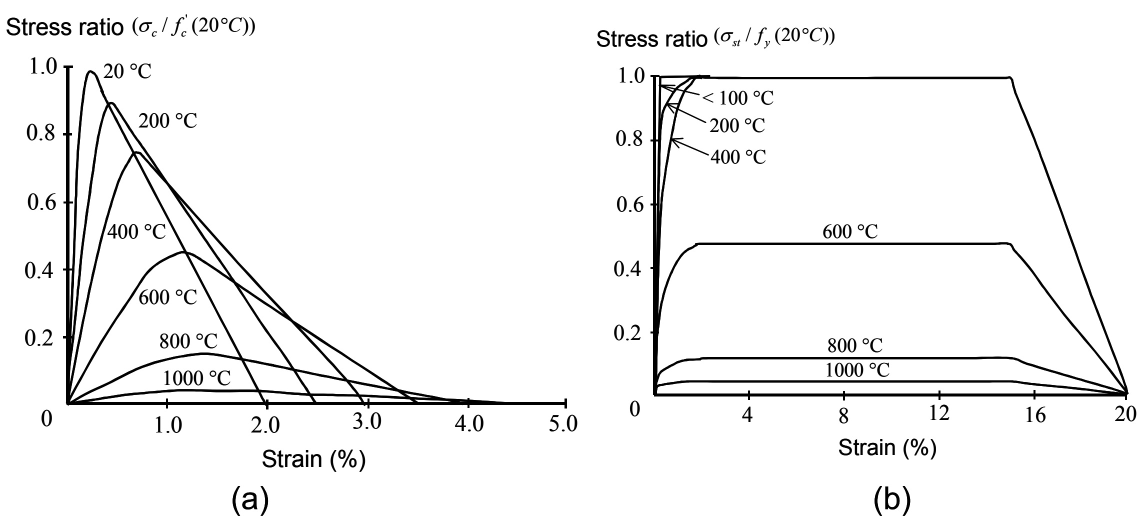

FIGURE 2: Stress-strain relationships of concrete and steel at elevated temperature: (a) Concrete in compression; (b) Steel.

$$\mathbf{D}_c^{'} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mu G_c & 0 \\ 0 & 0 & \mu G_c \end{bmatrix} \quad (8)$$

where G_c is the shear modulus of concrete and μ is the shear retention factor for which $0<\mu\leq 1.0$. After crushing, the concrete is assumed to lose all strength and stiffness.

The model described above has been incorporated within *Vulcan* in order to model the structural behaviour of steel, reinforced concrete and composite buildings in fire conditions. The total loading or temperature rise for which the response of the structure is to be traced is divided into a number of steps. It is assumed that changes in the loads or temperatures occur only at the beginning or end of a step. During any step the external loads and temperatures in the segments of all elements are assumed to remain constant.

ILLUSTRATIVE STUDIES

A simple example study is now presented in order to illustrate the capabilities of the new beam-column element. The objective is to show the considerable improvement in accuracy given by the new formulation at very high deflections. This is followed by a comparison against the record from a full-scale fire test on a composite beam [15].

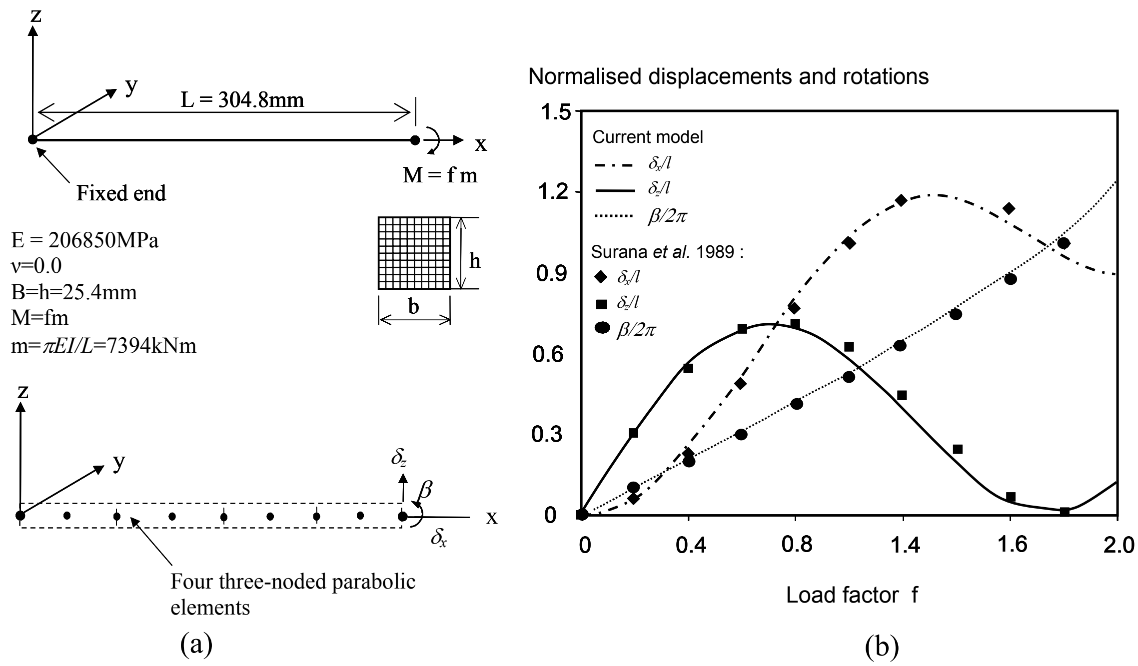

FIGURE 3: Cantilever beam subjected to a concentrated end moment: (a) Cantilever beam and its finite element model; (b) Load-deflection behaviour.

A cantilever beam of square cross-section composed of uniform linear-elastic material, subjected to a concentrated end moment *M,* as shown in Fig. 3(a), was analysed. In this analysis the cross-section of the beam was divided into 100 segments. Fig. 3(b) shows a plot of horizontal and vertical end-displacements δ_x, δ_y and end-rotation θ versus the load factor *f*, together with the results from Surana and Sorem [16]. It is evident that there is a very good agreement between the current model and these results, showing the capability of this formulation to deal with very high displacements, considerably beyond the range which would be encountered in real structural fire engineering examples.

A full-scale ISO384 standard fire test on a "*Slimflor*" beam (see Fig. 4(a)), which had been tested at Warrington Fire Research Centre in 1996 [15], was modelled using the new beam element. The test specimen consisted of a simply supported composite 280ASB100 asymmetric beam with normal-weight concrete cast onto deep-deck profiled sheeting with an A142 mesh. The span of the beam was 4.5m and four hydraulic rams were used to apply a total load of 338kN (84.6kN each), to the test assembly. The load was applied directly to the upper flange of the steel section at positions corresponding to the 1/8, 3/8, 5/8 and 7/8 points of the supported span. The self-weight of the beam was 4.81kN/m. Fig. 4(a) provides details of the specimens. The yield strength of the steel at ambient temperature was measured as 402MPa and this was used in the analysis. Because no test data were available on the compressive strength of the concrete or the yield strength of reinforcement these were respectively assumed at room temperature to be 25MPa and 460MPa in the modelling.

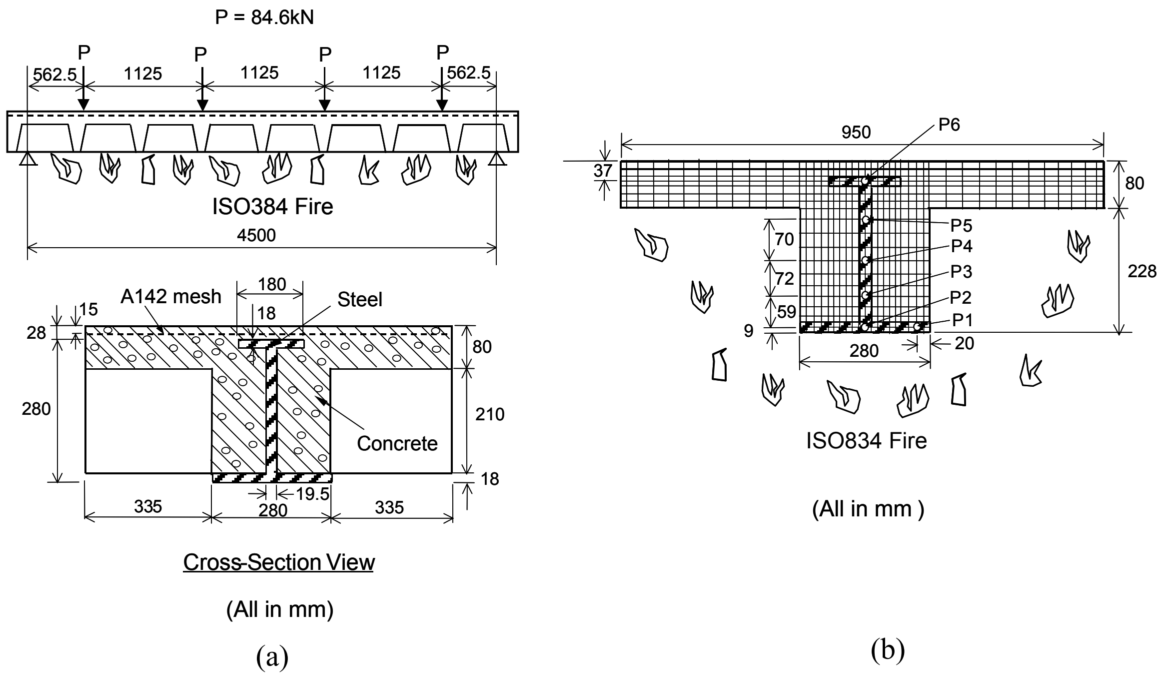

FIGURE 4: A full-scale ISO384 standard fire test on a "*Slimflor*" beam: (a) Details of the *Slimflor* beam tested; (b) The element segmentation mesh adopted.

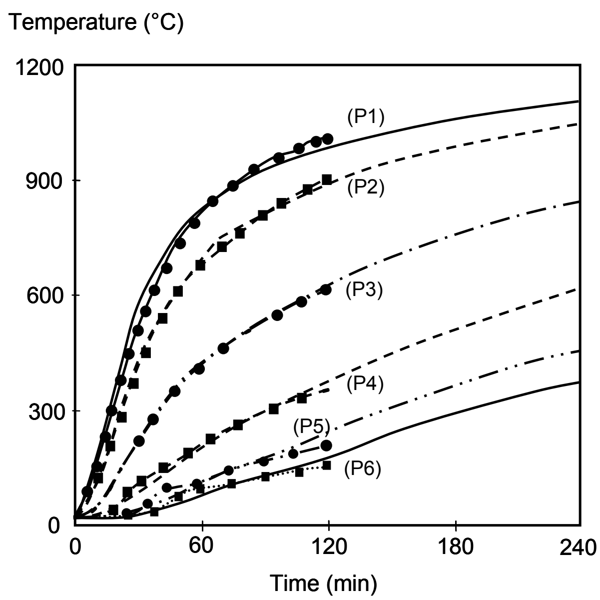

FIGURE 5: Comparison of predicted and measured [15] temperatures at several positions of steelwork of the cross-section of the *Slimflor* beam.

A large number of thermocouples were employed to measure the temperatures within the steelwork and concrete during the test. However, from the test data it is quite difficult to specify reasonably the temperatures of all the segments within the cross-section of the composite beam. The first step of the analysis was therefore to perform a thermal analysis on the structure. *Vulcan* has recently been extended by one of the authors to include a two-dimensional non-linear finite element procedure to predict the temperature distributions within the cross-sections of structural member subject to given fire time-temperature regimes. This is largely based on previous work [17] by the first author. The thermal properties of the steel and concrete are assumed to change with temperature, and the influence of moisture initially held within the concrete and protection materials is taken into account in their properties. The finite element mesh used to calculate the temperature distribution in the beam cross-section is shown in Fig. 4(b). In this analysis the thermal properties given in Eurocode 4 for concrete and steel were adopted. The predicted temperatures at various positions on the steelwork cross-section are shown in Fig. 5 together with the test results. It is evident that the predictions and test results are in quite good agreement.

The predicted temperatures were used as input to perform structural analysis for the test beam in this study. The mesh used in thermal analysis was also used for the segmentation of the cross-section of the composite beam. Hence, in the structural analysis the cross-section of the beam was divided into 380 steel or concrete segments and the slab was divided into 9 layers which included 7 concrete and 2 reinforcing steel layers. The predicted central deflection of the beam for this test is plotted against time in Fig. 6, together with test results. It is evident that the analytical predictions are in very good agreement with test results. The test was stopped before 120 min but the prediction is capable of extending the "test" to 240 min, albeit with very large deflection. From both the test and computer modelling it is clear that the concrete in the *Slimflor* system provides very effective insulation to the steel beam (see Fig. 5). The temperatures within the top half of the steel beam cross-section remain below 400°C at 120 minutes into the test. The temperature of the top flange is less than 200°C. This gives a considerable enhancement to the fire resistance of the structure.

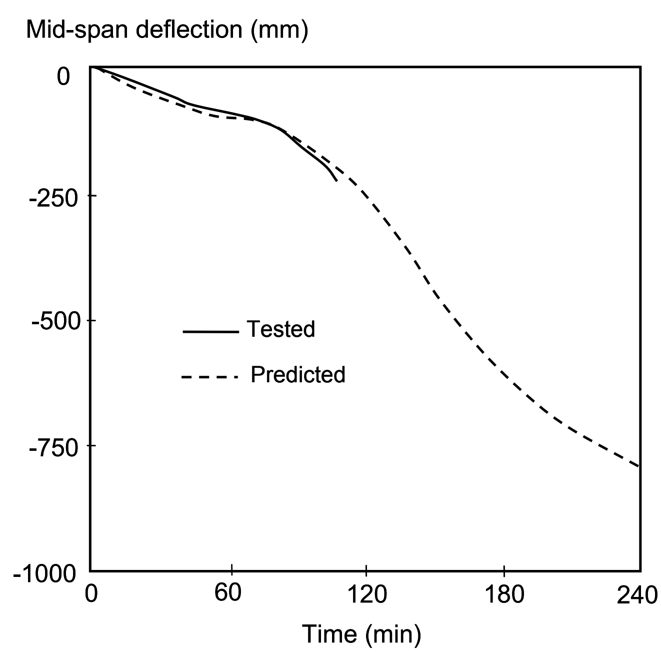

FIGURE 6: Comparison of predicted and measured [15] deflections at mid-span of the *Slimflor* beam.

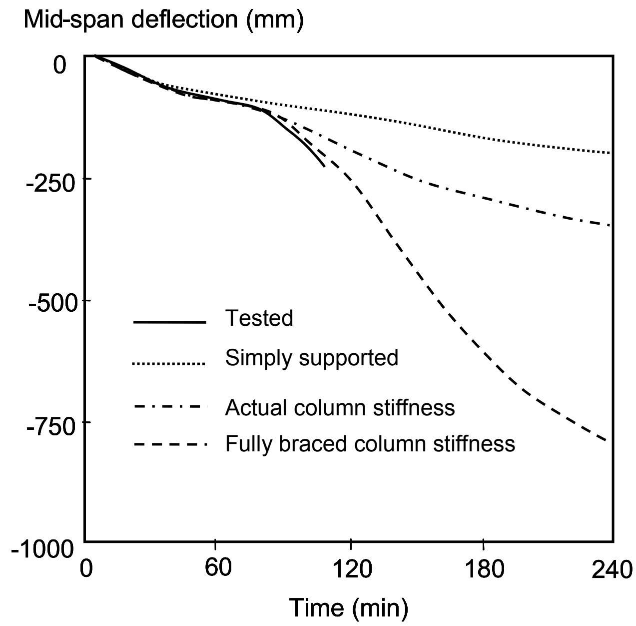

FIGURE 7: Predicted deflections at mid-span of the tested beam for different end support conditions.

In order to investigate the influences of catenary action on the behaviour of the tested beam an axial spring has been introduced to represent the column stiffness at one support of the beam. Two different stiffnesses have been considered. To represent a fully braced frame a very large value (10^{10} N/mm) is assumed, whilst for an unbraced frame stiffnesses were calculated assuming a column size of 305x305x198UC, calculated for a 10-storey building according to BS5950 Part 1 [18]. Assuming that the height of a storey is 4m then the equivalent spring stiffnesses are 5,630 N/mm for the column minor axes. Fig. 7 compares the deflections at the mid-span of the test beam for these different end support conditions. Also shown are the deflections for a simply supported condition, which provides no axial restraint, demonstrating that catenary action has a significant influence.

CONCLUSIONS

The model proposed in this paper is based on the formulations proposed by Bathe [10], but including further developments to enable the analysis of steel, reinforced concrete and composite beam-column framed structures subject to fire. In this formulation both geometric and material non-linearities have been taken into account. The cross-section of the beam-column element is divided into a number of segments, of which each can have different material, temperature and mechanical properties. The more complicated aspects of structural behaviour in fire conditions, such as thermal expansion, cracking or crushing of concrete, yielding of steel and change of material properties with temperature are modelled. In this study a Total Lagrangian approach has been adopted throughout, in which displacements are referred to the original configuration and small strains are assumed. Since the elements possess an offset capability, these elements can also be used, with shell or plate elements, for modelling of composite buildings in fire. One simple numerical example using linear elastic material has been presented to demonstrate the capability and the accuracy of the elements. It is evident that the proposed model produces logical results when the effects of geometric non-linearity are considered, even at unrealistically high deflections. Finally, a full-scale standard fire test on a *Slimflor* beam has been modelled to show the element's capabilities in modelling situations in which both steel and concrete are key structural components, and there is non-uniform heating by fire. It is evident that the non-linear procedure proposed in this paper is very suitable for modelling such situations in fire conditions. In the current model each node of the elements has six degree of freedom, three translational and three rotational in both local and global coordinates, and hence there is a considerable saving of processing time compared to the previous *Vulcan* beam-column formulation [5]. Since a proper geometric transformation is now possible between the local and global degrees of freedom, the previous difficulty in modelling of sloping members no longer exists.

ACKNOWLEDGMENT

The authors gratefully acknowledge the financial support of the Engineering and Physical Sciences Research Council of Great Britain, under Grant number GR/M99194.

REFERENCES

[1] British Standards Institution, *BS5950: Structural Use of Steelwork in Buildings: Part 8: Code of Practice for Fire Resistant Design*, BSI, London, 1990.

[2] European Committee for Standardisation, *Eurocode 3, Design of Steel Structures, Part 1.2: Structural Fire Design (Draft)*, Commission of the European Communities, Brussels, 1993.

[3] International Organisation for Standardisation, *ISO 834: Fire Resistance Tests - Elements of Building Construction,* 1985.

[4] Swinden Technology Centre , *The Behaviour of Multi-Storey Steel-Framed Buildings in Fire: A European Joint Research Programme*, British Steel plc, Rotherham, UK, 1999.

[5] Najjar S.R., Burgess I.W., A non-linear analysis for three-dimensional steel frames in fire conditions, *Engineering Structures*, 18(1), 1996, pp. 77-89.

[6] Bailey C.G., Burgess I.W. and Plank R.J., Computer simulation of a full-scale structural fire test, *The Structural Engineer,* 74 (6), 1996, pp. 93-100.

[7] Huang Z., Burgess I.W. and Plank R.J., Modelling membrane action of concrete slabs in composite buildings in fire. Part I: Theoretical development, *Journal of Structural Engineering,* ASCE, 129(8), 2003, pp.1093-1102.

[8] Huang Z., Burgess I.W. and Plank R.J., Modelling membrane action of concrete slabs in composite buildings in fire. Part II: Validations, *Journal of Structural Engineering,* ASCE, 129(8), 2003, pp. 1103-1112.

[9] Wong S.Y.,), *The structural response of industrial portal frame structures in fire*, PhD Thesis, University of Sheffield, 2001.

[10] Bathe K.J., *Finite element procedures in engineering analysis*, Prentice-Hall Inc., New Jersey, 1982.

[11] Bathe K.J., *Finite element procedures*, Prentice-Hall Inc., New Jersey, 1996.

[12] Huang Z., Burgess I.W. and Plank R.J., *A non-linear beam-column element for 3D modelling of general cross-sections in fire,* Research Report DCSE/03/F/1, Department of Civil & Structural Engineering, University of Sheffield, 2003.

[13] European Committee for Standardisation *Eurocode 4, Design of Composite Steel and Concrete Structures, Part 1.2: Structural Fire Design (Draft)*, Commission of the European Communities, Brussels, 1992.

[14] American Society of Civil Engineers, *Finite element analysis of reinforced concrete*, New York, 1982.

[15] Wainman D.E. and Martin D.M., *Preliminary assessment of the data arising from a standard fire resistance test performed on a Slimflor beam at the Warrington Fire Research Centre on 14th February, 1996*, Technical Note, SL/HED/TN/S2440/4/96/D, British Steel plc, Swinden Technology Centre, UK, 1996.

[16] Surana K. and Sorem R., Geometrically non-linear formulation for three dimensional curved beam elements with large rotations, *International Journal for Numerical Methods in Engineering*, 28, 1989, pp. 43-73.

[17] Huang Z., Platten A. and Roberts J., Non-linear finite element model to predict temperature histories within reinforced concrete in fires, *Building and Environment*, 31(2), 1996, pp. 109-118.

[18] British Standards Institution, *BS5950: Structural Use of Steelwork in Buildings: Part 1: Code of Practice for Design: rolled and welded sections*, London, 2000.