MODELLING MEMBRANE ACTION OF MODEL-SCALE SLABS AT AMBIENT AND ELEVATED TEMPERATURES

S. J. FOSTER[1], I. W. BURGESS[2] and R.PLANK[3]

ABSTRACT

This paper presents the results from a series of numerical studies conducted on unrestrained reinforced slabs, of relatively high span/depth ratio, at large displacements. Previous experimental work involved a series of tests carried out on thin model-scale slabs at ambient and elevated temperatures. The purpose of the studies reported here was to investigate the influence of the following parameters, at both ambient and elevated temperatures, on the degree of mobilisation of membrane action within the model scale slabs:

- Thermal gradient;
- Bond strength;
- Distribution and cross sectional area of reinforcement;
- The influence of concrete tension;

The more detailed issues being studied include the major factors influencing the development of the tensile membrane forces within the slabs at both ambient and elevated temperatures. Numerical and experimental results are compared with the well known BRE Method, which is based on a simplified representation of tensile membrane action. The results of these comparisons show that the latter is generally conservative, although the approach is an upper-bound one and may not be applicable under all circumstances.

KEYWORDS: Tensile membrane action, concrete, slabs, large displacements, high temperature experiments, fire performance, yield line theory.

1. INTRODUCTION

There has been a great deal of work in the last decade in modelling full-scale composite frames in fire conditions. However, there has been limited investigative work carried out on the structural behaviour of floor systems at large displacements at both ambient

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[1] Ove Arup Ltd, 30 Barkers Pool, Sheffield, S1 3HB, UK; samantha.foster@arup.com

[2] Department of Civil & Structural Engineering, University of Sheffield, Sheffield, S1 3JD, UK; ian.burgess@sheffield.ac.uk

[3] School of Architectural Studies, University of Sheffield, S10 2TN, UK; r.j.plank@sheffield.ac.uk

and elevated temperatures. Early investigative work on large displacement of slabs was undertaken in the late 1960s, and this showed that at ambient temperature reinforced concrete slabs perform better than is conventionally thought. The increased load capacity of the slab was attributed to in-plane forces which are created due to the boundary conditions and resultant displacements. The slab becomes a self equilibrating system. This mechanism is known as Tensile Membrane Action as the slab is resisting the loads by diaphragm action.

Work has been undertaken to investigate the degree of mobilisation of tensile membrane action within a slab at both ambient and elevated temperatures due to a number of parameters:

- Bond strength,
- Distribution and cross sectional area of reinforcement,
- The influence of concrete tension,
- Influence of thermal gradient.

Some comparisons have been made with the well known BRE Method, which is based on a simplified representation of Tensile Membrane Action [1].

1. MODEL SET-UP

Modelling of the behaviour of the slabs at ambient and elevated temperature was undertaken using the finite element software *Vulcan*. This software has been extensively validated over the years with experimental data collated from the full-scale tests carried out at Cardington. *Vulcan* models the complex factors that arise in fire conditions, such as thermal expansion of the elements, change of material properties with temperature and concrete cracking and crushing. Fig. 1 shows how the slab elements are configured.

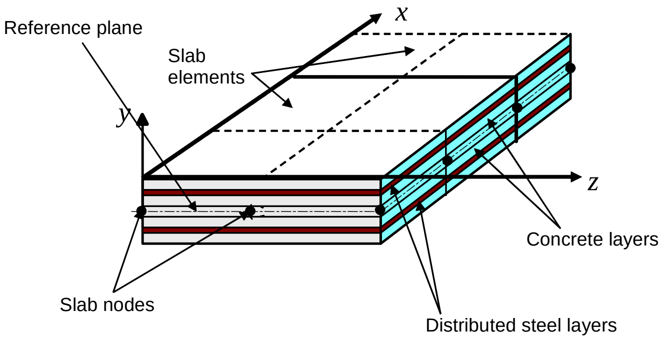


Fig. 1 – The distribution of slab elements

The main features of the program are not discussed here but further details are given in references [2, 3].

2.1 Validation of Model

The model-scale slabs were representative of tested slabs that have been tested at both ambient and elevated temperatures. The models were created using an assemblage of finite plate elements using a quadrilateral 9-noded element. The elements were divided into several layers representing the concrete and reinforcing steel (Fig. 1). The tested slabs were horizontally unrestrained with edges simply supported. In order to check that the boundary conditions of modelled slabs were representative of the tested slabs, crack patterns for the bottom layer of the slab (Figs. 2(a) and 2(b)) were analysed. The critical load intensity in tests was approximately 3.3kN/m^2, which compared well to the 3.6kN/m^2 predicted by *Vulcan* for the initiation of cracking. Beyond this stage it can be seen that the crack patterns show a developing yield-line mechanism.

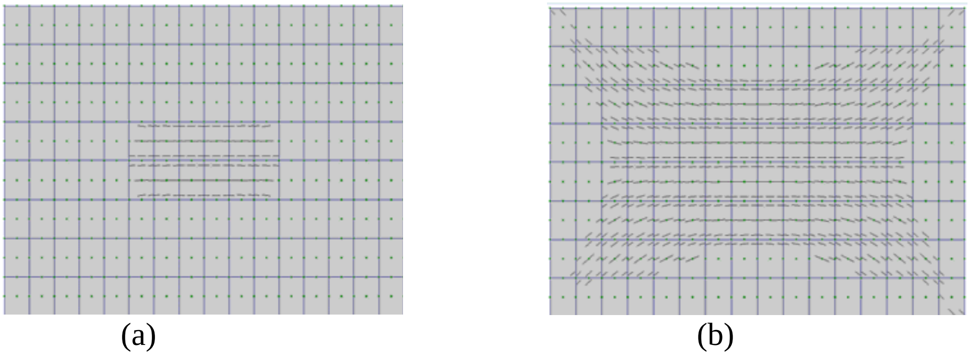

(a) (b)

Fig. 2 - Crack patterns for bottom layer of full slab for load (a) 3.74kN/m^2 and (b) 4.96kN/m^2

2. PARAMETRIC STUDIES AT AMBIENT TEMPERATURE

Several experimental studies were undertaken at ambient temperature to investigate the influence of the following parameters on the load capacity of the slab:

- Bond strength
- Development of membrane forces

Further parametric studies were conducted using *Vulcan* to study the influence of concrete tension and reinforcement yield strength on the slab behaviour. The influence of concrete tension could not be evaluated experimentally. The experimental and numerical results were also compared with the BRE Simple Design Method.

The model scale slabs in the ambient tests had a total reinforcement of 0.3% of the slab cross-sectional area. In order to investigate the influence of the bond strength on the overall load carrying capacity of the slab, the tested slabs were reinforced with two types of wire; smooth and deformed wire. The tested ductility was 20% for the smooth wire and 11% for the deformed wire. Table 1 shows a comparison of two tested slabs and the material data collected. The tests are identical in that the slabs have the same aspect ratio and arrangement of reinforcement. The main difference between these two tests is the reinforcement type, Test 1 was reinforced with smooth wire, and Test 2 with deformed wire. These slabs had isotropic reinforcement in that they had the same steel area in both the short and long spans.

Table 1. Isotropic slabs of aspect ratio 1.55

Test	Size (mm)	t(mm)	f_y(N/mm^2)	f_c(N/mm^2)	f_t(N/mm^2)	Yield line capacity(kN/m^2)
1 *smooth wire*	850x550	19.5	256	28	1.76	2.54
2 *Deformed wire*	850x550	16	248	25	1.66	1.99

The load carrying capacities of the two slabs are shown in Fig. 3, and are shown as enhancements above yield-line capacity to illustrate the development of tensile membrane action within the slabs. The experimental results have also been compared with theoretical predictions by the simplified approach; which are shown in Fig. 3 by the two straight lines. The load carrying capacity of the two slabs quickly increases with increasing slab displacement up to a vertical displacement of 30mm, beyond which the actual capacity of the slab in Test 2 decreases. The reason for this reduction in capacity was fracture of the deformed wire. Both slabs had suffered cracking, but the slab reinforced with smooth wire was still integral when removed from the loading rig whereas for that with deformed wire the fracture of the reinforcement was clearly seen.

The test results of the slab with the smooth wire compare well to the theoretical predictions, since, due to their lower bond strength and greater ductility, the wires did not fracture. The test results of the slab with deformed wire did not compare well to theory, as the analysis predicted an ever-increasing load capacity, because the method does not take into consideration the decrease in load due to fracture of reinforcement.

Comparison of the test results with *Vulcan* is also shown in Fig. 3. The numerical results in which concrete tensile strength is accounted for show predictions in the region of 1.6 times the experimental initial peak, followed by massive amounts of simultaneous cracking which causes the apparent “plateaux” in the strength of these very lightly reinforced slabs. For comparison purposes it is more instructive to inspect the modelling without including concrete tension. These curves show good agreement with both the simplified method and the tests.

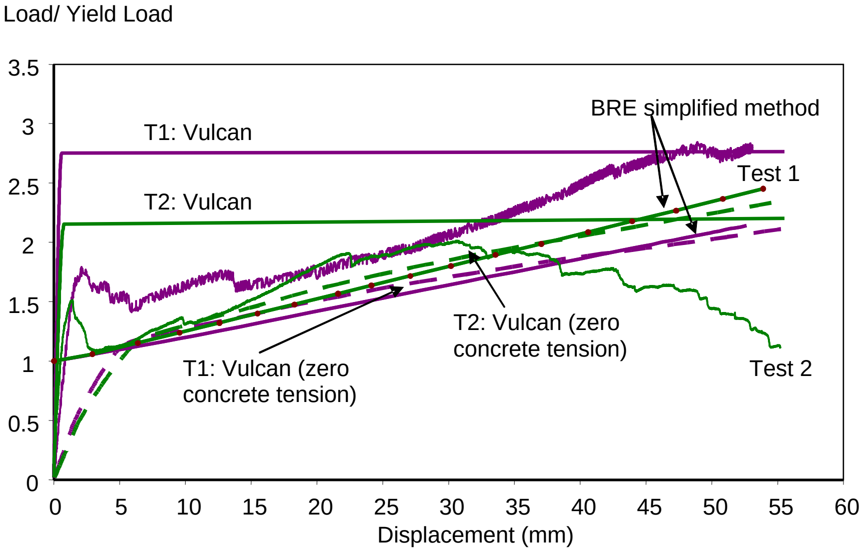


Fig. 3 - Comparison of Test 1 and Test 2 with *Vulcan* and simplified method

A study was undertaken to investigate the effect of variation of the yield strength of reinforcement and the compressive and tensile strengths of concrete. A range of concrete strengths were investigated, from very low (F_c=6N/mm^2) to the strength measured on the day of the test. The influence of zero concrete tensile strength was investigated. The results of this study are presented in Fig. 4. The study showed that;

- In initial deflection behaviour, the concrete tensile strength is very important,
- At large deflections, the tensile strength of the reinforcement is important in determining the load capacity of the slab.

The load-displacement curves for various concrete strengths shown in Fig. 4 converge at large displacement, indicating that concrete strength does not affect the enhancement factor.

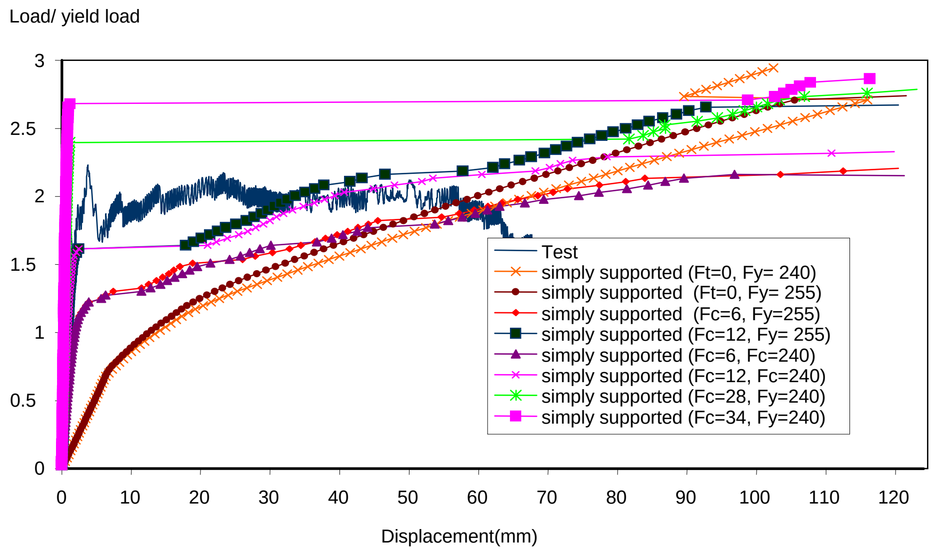


Fig. 4 - Investigation of the influence of the yield strength and concrete compressive for slab of aspect ratio 1.55

The development of membrane forces within an isotropic slab (Test 1) at a mid-span displacement of 33mm and load of 3kN/m^2 is shown in Fig. 5. Compression forces are seen to form in the central zone of the quarter-slab, parallel to the short span (indicated by the blue vectors). The edges parallel to the long span move inwards, and as the slab displaces these compressive forces start to develop in the central zone as the slab starts to 'jam'. The zone of pure tension is represented by the thin red lines.

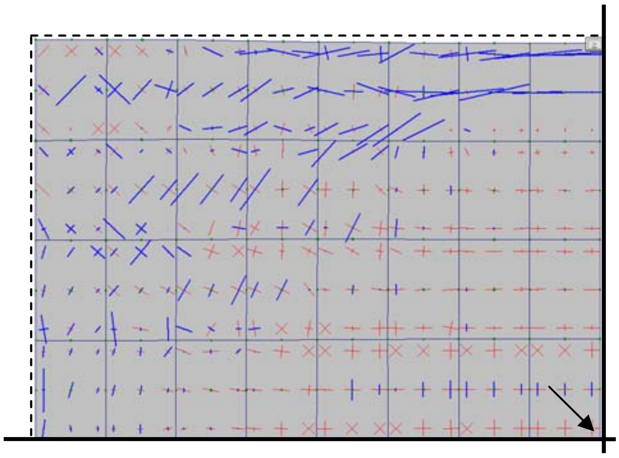


Fig. 5 - Membrane tractions for quarter-slab at 33mm deflection for a load of 3kN/m^2 and zero concrete tensile strength

This 'jamming' of sections of the slab was observed during testing, and therefore the predictions by *Vulcan* seem to tie in well to the observed slab behaviour. However, the simplified method does not consider the development of compressive forces within the central zone. Further investigations have been undertaken to assess the sensitivity of the enhancement of load capacity to these forces, but this is difficult to quantify as it occurs once the slab has formed its yield line mechanism and is at large displacement.

Numerical studies were also performed to investigate the slab's response to increased areas of total reinforcement and to compare this with the simplified design method. Investigations were undertaken on model slabs with cross-sectional areas of reinforcement varying from 0.05% to 0.7% whilst other variables remained constant. The enhancements predicted by the simplified method are shown in Fig. 6 and by numerical analysis in Fig. 7.

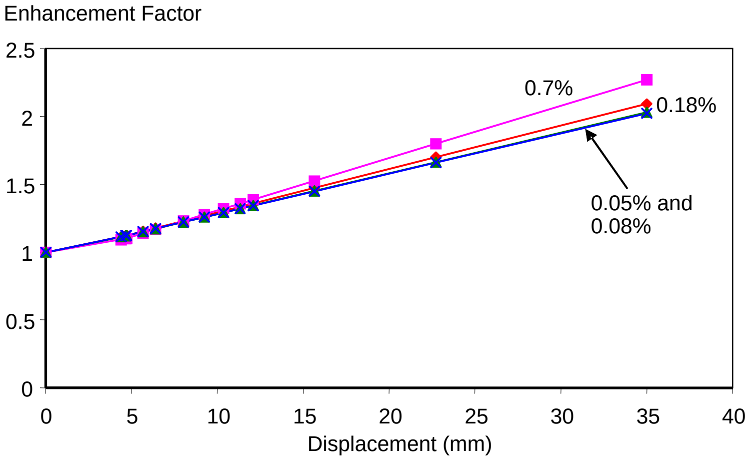


Fig. 6 -Enhancements predicted by the Simple Design Method for various reinforcement percentages.

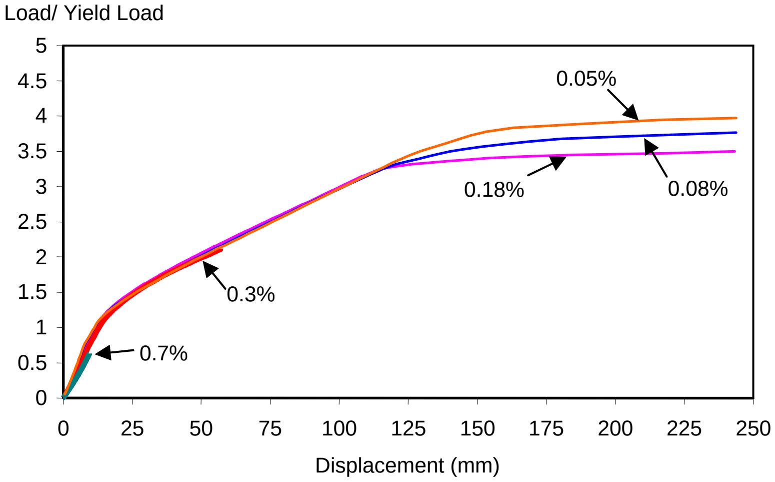


Fig. 7-Enhancements predicted by *Vulcan* without concrete tension.

The results highlight the fact that the numerical predictions do not follow the same pattern as the Simplified Method's predictions. The latter limits the amount of reinforcement to 0.7%, because of the stress block adopted. The approach is based on yield line theory, and so slabs must have high span/depth ratios, have low reinforcement percentages and be designed as 'under-reinforced'. Cross-sections must be ductile, and limitations have been put on the percentage of reinforcement, with 0.75% being the maximum allowable for yield line design [4].

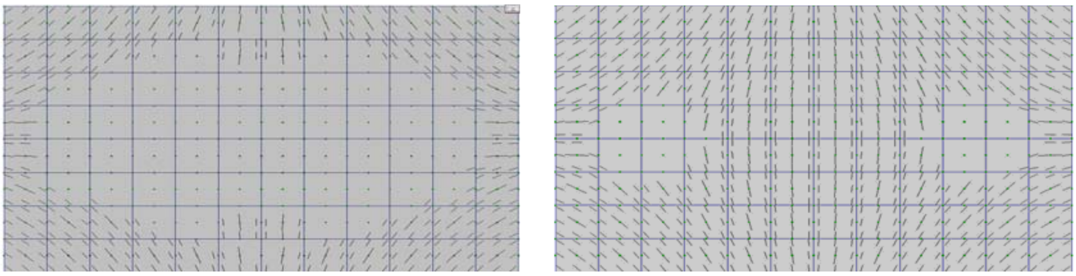
3. PARAMETRIC STUDIES AT ELEVATED TEMPERATURES

A number of experimental studies were undertaken at elevated temperatures to investigate the influence of the following parameters on the load capacity of the slab:

- Bond strength,
- Thermal gradient,
- Reinforcement area ,
- Development of membrane forces .

Further parameters were studied using *Vulcan*. These included the influence of concrete tension and the development of membrane forces within the slab at elevated temperatures. The experimental and numerical results were also compared with the Simplified Design Method.

The crack patterns on the bottom surface of the slab were plotted at various temperatures (see Fig. 8). It can be seen that the crack initiation is different from that at ambient temperatures, with cracks developing at the corners of the slab followed by transverse cracking across the short span of the slab at higher temperatures.



99°C 153°C

Fig. 8 - Crack patterns for full slab at different bottom surface temperatures.

The influence of bond strength was investigated experimentally by using smooth and deformed wires within the slab. Fig.9 shows comparisons between two tests with identical conditions apart from the reinforcement type. Test 4 was reinforced with smooth wire and Test 3 with deformed wire. The slabs had a total reinforcement percentage of 0.15% and load ratio of 1.7. The *Vulcan* predictions for both tests showed a good degree of accuracy, with slight discrepancies at temperatures in excess of 600°C. The *Vulcan* predictions towards the end of Test 4 are in close agreement with the measured mid-span displacements. However towards the end of Test 3, the *Vulcan* predictions diverge from the test results. Further studies indicated that bond between the reinforcement and concrete as being lost at this stage. *Vulcan* can not at present model this effect.

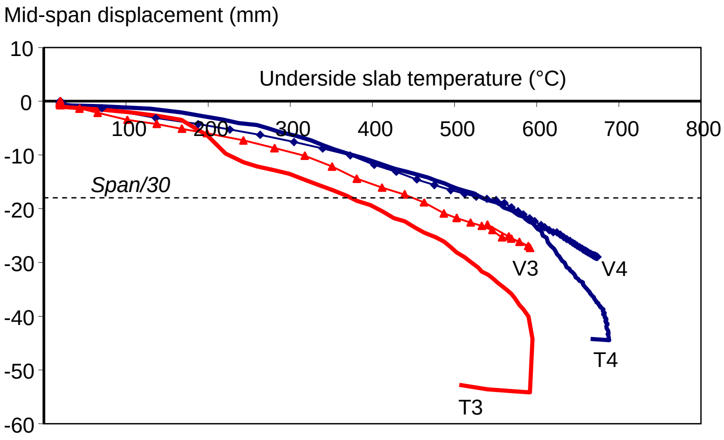


Fig. 9 - Comparison of mid-span displacements for Tests 3 and 4 with *Vulcan* predictions.

Numerical studies were undertaken to investigate the effects of temperature profile on the membrane action of the slabs. The test data is shown in Table 2.

Table 2: Test 5 parameters.

Test 5	Reinf.Area(%)	F_c (N/mm^2)	Yield Capacity (kN/m^2)	Load Ratio Q/W_u
Smooth Wire	0.15	44	2.21	2.8

Fig. 10 shows the resulting vertical displacements, assuming different linear temperature gradients across the slab depth.

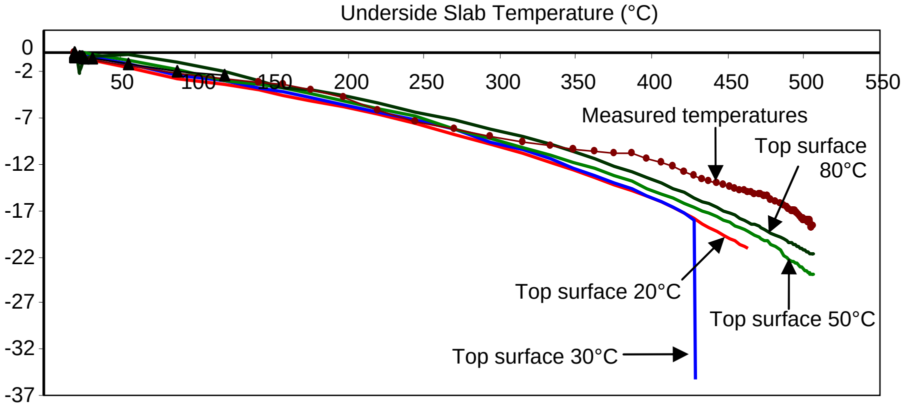


Fig. 10 - Comparison of mid-span displacements with Vulcan predictions for various linear temperature gradients.

For lower surface temperatures in the range 20°C-300°C, the rates of displacement for all temperature profiles were similar. Above 300°C the rate of displacement for the measured temperature profile decreased compared to the other temperature profiles. At the beginning of a test, high thermal gradients are created through the depth at a time when strength and stiffness are reasonably intact. These gradients reduce as the top surface temperature of the slab increases, as lower surface stiffness and strength is becoming considerably degraded.

Studies were carried out on the effects of reinforcement area on slab performance. The results indicated that slabs reinforced with lower reinforcement performed better than those with high percentages. Fig.11 shows the test results and Fig.12 shows a comparison with the BRE Simplified Design Method.

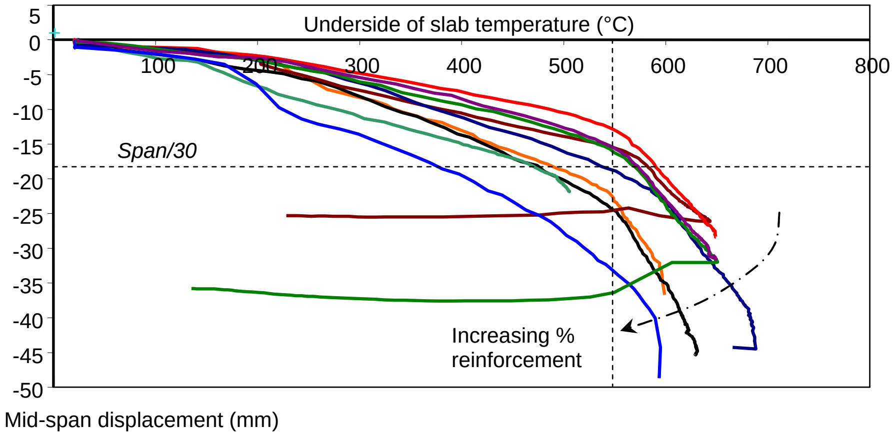


Fig. 11- Comparison of Test results with increasing slab reinforcement areas.

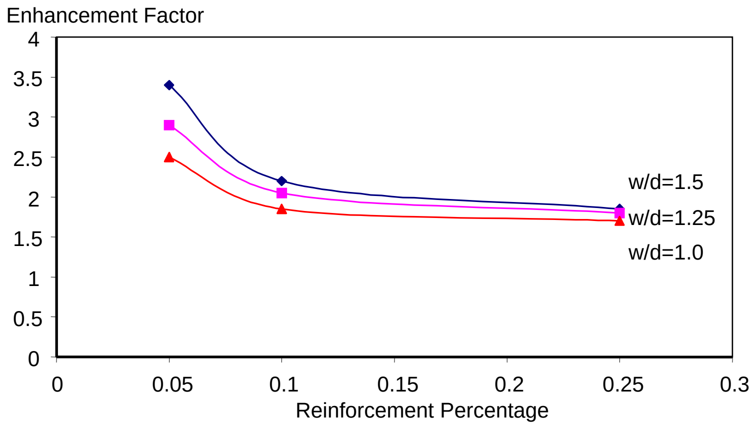
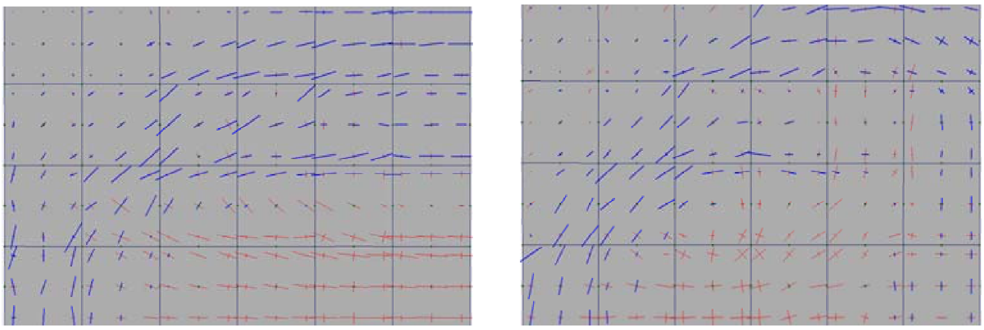


Fig. 12- Comparison of the applied load/yield load ratio with percentage reinforcement with displacement/effective depth (w/d)

Observing the development of membrane forces shows that for slab displacements between 10-15mm, compressive forces are seen to develop across the short span of the slab, and these are then relieved at larger displacements as the slab develops membrane action, with tensile forces within the central zone and a compressive peripheral ring around the edges of the slab. The compressive band seems to be influenced by the area of reinforcement. Figs. 13(a) and 13(b) compare the membrane traction plots for slabs which have different reinforcement percentages. The development of membrane forces at high temperatures is different from that at ambient temperature. Further investigations have identified that the geometry of the slab also influences the development of membrane forces.



(a) (b)

Fig. 13 - Membrane traction plots for (a) slab reinforced with 0.5% reinforcement at 221°C and (b) 0.1% reinforcement at 341°C (bottom surface temperatures).

Studies were undertaken to investigate the slab behaviour with no contribution from concrete tensile strength or stiffness. This is one of the most important factors in determining the overall slab behaviour and is probably the hardest part to quantify. Slabs which have undergone extensive cracking will have little contribution from concrete in tension. However, slabs which have undergone little or no cracking will gain some strength contribution from concrete in tension. Fig. 14 shows that the influence of concrete in tension does not become significant until after 200°C, and that the thermal gradient is the most influential factor in initial displacements. Fig. 15 illustrates that as the percentage of reinforcement increases the influence of concrete in tension diminishes.

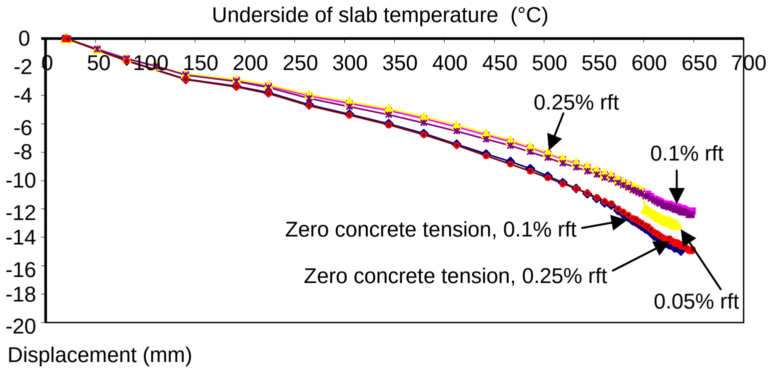


Fig. 14 - Numerical study comparing the displacements of models with varying amounts of reinforcement at zero applied load.

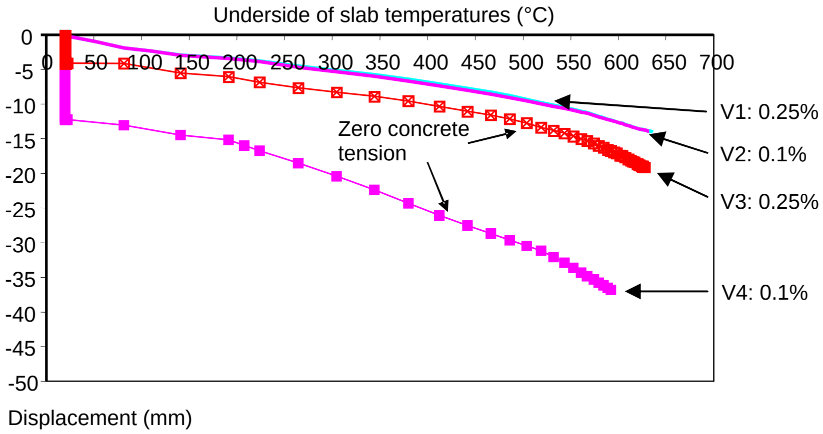


Fig. 15 - Numerical study comparing loaded slabs with varying areas of reinforcement with and without concrete tension.

5. COMPARISONS OF SLAB BEHAVIOUR AT AMBIENT AND ELEVATED TEMPERATURES WITH DIFFERENT FACTORS

From the detailed investigations undertaken the performance of loaded reinforced concrete slabs is seen to differ at elevated temperatures from that at ambient temperature. At ambient temperature, the main mechanisms that develop are:

- Development of yield line cracks, with the reinforcement yielding along these cracks.
- Concrete strength is not a significant factor at high displacements, but the strength and type of rebar used significantly affects the load capacity of the slab.
- The BRE Simplified Design Method is only applicable for slabs with low reinforcement percentages and high span/depth ratios.
- The tensile strength of concrete is significant at low deflections, but is very difficult to determine. The BRE Simplified Design Method does not consider the contribution of concrete tension within its approach.
- The simplified approach does not consider the effects of bond and fracture of reinforcement. It predicts continuously increasing linear enhancements.

- The work has highlighted the fact that *Vulcan* is currently unable to model reinforcement bond slip, and a criterion will need to be developed.

The high-temperature testing and numerical work on loaded reinforced concrete slabs has shown that slabs perform differently under heated and ambient-temperature conditions. The main mechanisms that develop at high temperatures are:

- At low displacements, thermal bowing of the slab is a significant factor in the slab displacements.
- At large displacements, concrete tensile strength is the main factor in the contribution of the slabs' resistance to the applied load.
- There was a difference in load-carrying capacity of slabs reinforced with smooth and deformed wire. However, this may not be entirely due to the reinforcement type as its influence is difficult to isolate.
- The development of membrane forces at high temperatures is different from their development at ambient temperature, and seems to be dependent on the aspect ratio and percentage of reinforcement.

At ambient temperature, the influence of concrete tension contributed to the initial load carrying capacity of the slab. The results from the high-temperature tests showed that up to 200°C, the thermal gradient and expansion of the slab is the main dominant factor. Above 200°C there is an increasing contribution by concrete tensile strength.

6. CONCLUSIONS

From the studies undertaken it has been shown that the performance of slabs at ambient temperature is very different from that at elevated temperatures. Some of main factors influencing the slab's performance at ambient and elevated temperatures are:

- Bond strength and type of reinforcement within the slab,
- Concrete tensile strength,
- Thermal bowing,
- Reinforcement percentage,
- Aspect ratio of the slab.

It is apparent from the tests that there is a lack of published test data for heated reinforced concrete slabs. The tests undertaken were limited to simple support conditions, and further testing is required on different floor systems and boundary conditions.

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