EFFECT OF TRANSIENT THERMAL STRAIN ON THE BUCKLING OF SLENDER CONCRETE AND CONCRETE-FILLED COLUMNS IN FIRE

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ABSTRACT

Pre-compressed concrete has been observed to acquire a large amount of irreversible strain (called Transient Thermal Strain or Load-Induced Thermal Strain) when it is heated. This effect appears not to occur when heating precedes the application of compressive stress. The objective of the research reported in this paper is to assess how this phenomenon affects the buckling resistance of slender concrete and concrete-filled hollow-section columns in fire. Preliminary analyses presented in the paper lead to the conclusion that TTS does have an adverse effect on the buckling temperatures of uniformly heated slender concrete columns.

1. INTRODUTION

Slender concrete and concrete-filled columns have recently been increasingly used, especially in non-seismic regions such as the UK (Fig. 1). Concrete, the main material of such columns, has a relatively newly-found property at high temperatures, defined either as Transient Thermal Strain (TTS) or as Load-Induced Thermal Strain (LITS) [1-6]. Both of these definitions describe the same phenomenon, in which pre-compressed concrete experiences a much larger compressive strain after heating than when it is loaded after being pre-heated, as shown in Fig. 2. This additional strain is large in magnitude and is not recoverable. However,

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it is neither clearly acknowledged in the Eurocodes [7] nor considered in the majority of structural analyses. Since concrete columns which are subjected to an accidental fire are nearly always pre-compressed when they are heated, they are a good example of structural elements which are vulnerable to TTS. The nature of TTS determines its significance. In stocky columns, it causes considerable extra contraction across the whole cross-section, and this has been observed in many tests on short cylinders. However, as the slenderness of the columns increases, such tests become less relevant as the failure mode switches from material crushing to lateral buckling. The way in which TTS affects buckling in such cases forms a gap in current knowledge which needs to be investigated.

Fig. 1 - External application of slender concrete-filled steel columns © *Stahl* + *Verbundbau*.

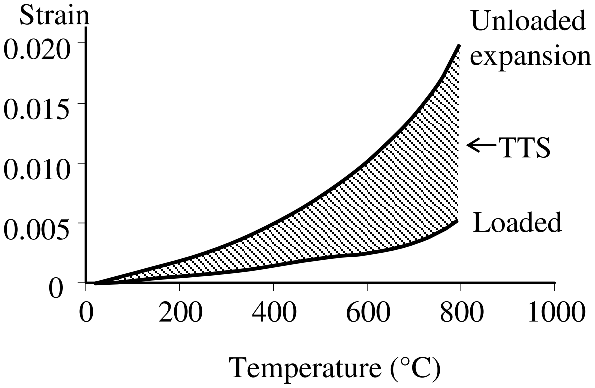


Fig. 2 – TTS, as the difference between total strain of pre-loaded and unloaded concrete, heated for the first time.

The classic simplified model due to F.R. Shanley [8], of a column buckling in its inelastic range, has been extended and its characteristics coded in an initial study of the mechanics of inelastic buckling. High-temperature analysis has also been conducted on this model, and several high-temperature concrete material models involving TTS have been compared. This has been followed by complementary FE analysis, developing the software *Vulcan*. Both analyses show a remarkable reduction of the buckling resistance of uniformly heated concrete columns caused by TTS, revealing the potential dangers of neglecting it in structural analysis and design. The effects of thermal gradients through the cross-sections of columns due to rapid heating, both symmetric and asymmetric, have also been considered.

Parametric studies on both concrete and concrete-filled columns concerning the effects of slenderness ratio, reinforcement and the steel casing are being carried out. The FE analyses using *Vulcan* will then be compared with test results and current design methods. The investigation will later move on to the effect of TTS on columns in continuous structures rather than columns in isolation.

2. SHANLEY-LIKE SIMPLIFIED COLUMN MODEL

In this research, concrete and concrete-filled columns, whose slenderness lies in the 'intermediate' range and so whose global failure mode is inelastic buckling, are of interest. Various inelastic buckling theories have been published since the late 1880s, including the tangent-modulus theory, the reduced-modulus theory and Shanley's theory [8-10]. Although in practice engineers tend to use the over-conservative tangent-modulus theory to obtain simple solutions to inelastic buckling problems, theoretically only Shanley's theory correctly describes the mechanics of inelastic buckling. Shanley demonstrates his theory with a simplified column model consisting of two rigid legs and an elastic-plastic hinge composed of two axial elements [8]. This model has been modified and extended as shown in Fig. 3, and its characteristics coded in this study. It has two degrees of freedom:

- Vertical movement u, which is the mean vertical movement of the springs;
- Rotation θ, which is proportional to the differential displacement of the extreme springs.

Since inelastic buckling is significantly rate-related, two dampers, one vertical and one rotational, are added to the original model. They respectively control the rates of increase of the two degrees of freedom u and θ. By changing the values of the two damping coefficients the extreme situations of inelastic buckling (bifurcation at the tangent-modulus or reduced-modulus buckling loads) can be simulated.

The two-spring Shanley model is extended with multiple springs in order to take into account the material continuity through a cross-section. The axial deformation of each spring is consistent with the usual linear strain-gradient assumption, and hence the mean displacement of each pair of springs at the equivalent locations on opposite sides of the vertical axis is u, and their differential displacement is proportional to θ.

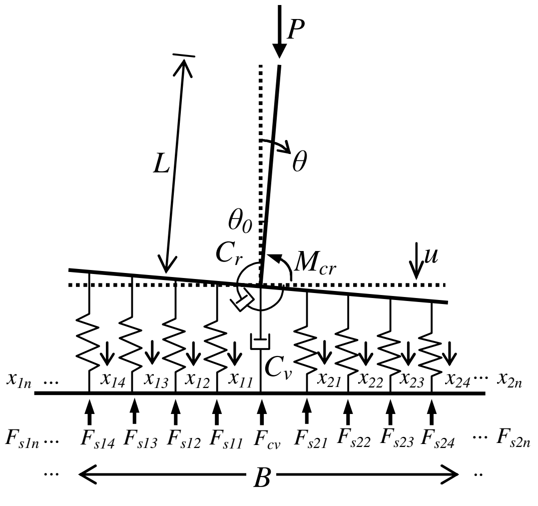


Fig. 3 – Multi-spring model.

2.1 Ambient-temperature analysis on Shanley-like model

A dynamic numerical analysis was conducted on the multi-spring model. Constant loading was imposed on the model. This simulated the application of a weight on top of the column, applied in a single step but very slowly, so that no initial velocity was induced. In the initial time step, the imbalance of the external and internal forces (whose difference is identical to the damping force) induces an acceleration, causing the model to move. The model continues to deform gradually through subsequent time steps until a new static equilibrium is reached, and this equilibrium position is recorded. Fig. 4 shows the development of the deformation of each spring through this dynamic procedure. The same procedure is repeated for successively higher loads until the rotation of the model is seen to diverge, indicating the final loss of stability by buckling. Plotting all the loads against the corresponding rotations θ, recorded at equilibrium, gives the full equilibrium path, as shown in Fig. 5.

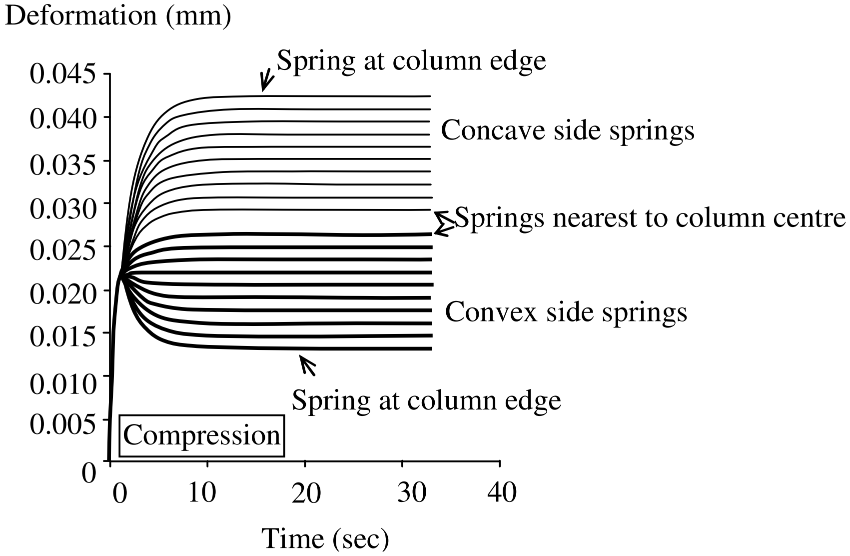


Fig. 4 – Development of the deformation of each spring over time under constant load in reaching equilibrium.

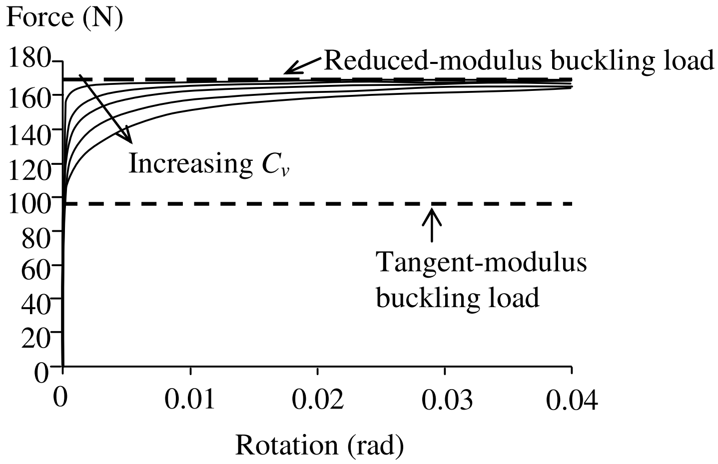


Fig. 5 – Buckling equilibrium load paths of the model with various damping ratios.

The equilibrium load paths of the model with various damping ratios are shown in Fig. 5. The analytical results are compared with the theoretical buckling loads: the tangent-modulus buckling load (the short-dashed straight line) and the reduced-modulus buckling load (the long-dashed straight line). Irrespective of the variation of damping, the rotation always starts to increase significantly at the tangent-modulus buckling load and then continues to increase as the force approaches the reduced-modulus buckling load, but the force never goes beyond this upper bound. This shows the column failing by buckling in the exact manner described by Shanley.

2.2 Elevated-temperature analysis on Shanley-like model

After the program describing the behaviour of the Shanley-like model had been evaluated at ambient temperature, it was then upgraded with Anderberg's [1] mathematical model of the material properties of concrete at elevated temperatures, in which the total strain of uniaxially compressed concrete subjected to elevated temperature consists of four components:

$$\varepsilon = \varepsilon_{th}(T) + \varepsilon_{\sigma}(\sigma) + \varepsilon_{tr}(T,\sigma) + \varepsilon_{cr}(T,\sigma,t) \tag{1}$$

in which ε = total strain; ε_{th} = thermal strain; ε_{σ} = instantaneous stress-induced strain; ε_{cr} = basic creep; ε_{tr} = transient thermal strain; T = temperature; σ = stress and t = time.

Instantaneous stress-induced strain ε_{σ} is the mechanical strain derived from the stress-strain curve. For any given temperature, ε_{σ} is only stress-dependent but the stress-strain curve varies with temperature. Transient thermal strain ε_{tr} is found to be reasonably linear with stress. It is also a nonlinear function of temperature and is approximately proportional to ε_{th}:

$$\varepsilon_{tr} = -k_{tr} \frac{\sigma}{\sigma_{u0}} \varepsilon_{th} \tag{2}$$

where k_{tr} is a constant whose value varies from 1.8 to 2.35, and σ_{u0} is the strength at 20°C.

Uniform temperature distribution across the springs was assumed. The loading procedure was upgraded from the constant loading used in the ambient-temperature analysis to step loading. Both steady-state and transient heating scenarios were applied. In the steady-state heating procedure, the step loading was applied at constant temperature until buckling occurred, and the buckling load of the column at this temperature was assessed. In the transient heating procedure, the column was loaded in steps to a certain load level, and then temperature was applied as a thermal loading, also step by step, until failure occurred. Obviously, the latter process more directly represents the real loading-heating situation of a column subject to an accidental fire, but it requires an instantaneous change of the material properties from a lower temperature to a higher one when the temperature changes, which causes considerable complexity in the numerical analysis. These two approaches were compared in order to assess the necessity of introducing this complexity into the research.

The results of the elevated-temperature analysis on an example model, including 20 springs which all have the same temperature, are briefly presented in Fig. 6. The two solid curves show the ultimate buckling loads of the model at various temperatures, with and without considering transient thermal strain. Comparing these shows that TTS can cause a remarkable reduction of the buckling resistance of slender concrete columns under uniform heating. This result is revealing, because the effect of TTS on buckling has hardly been considered in structural analysis, although the uniform-temperature assumption is very unrealistic.

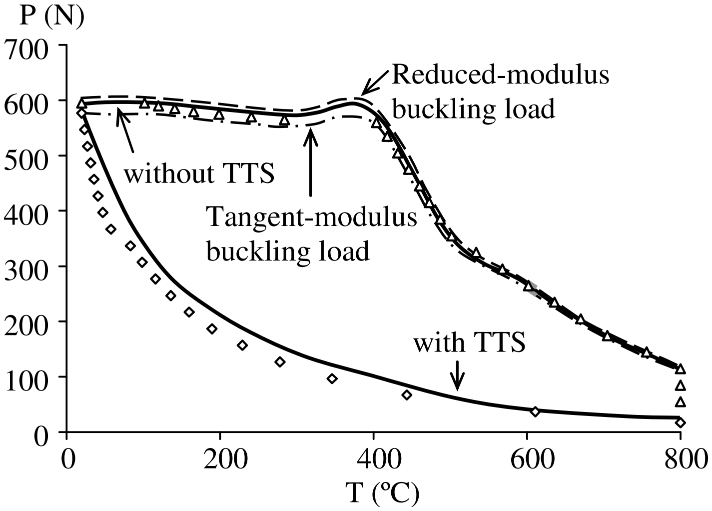


Fig. 6 – Buckling resistance of the model, with and without TTS, compared with theoretical buckling loads at elevated temperatures.

On the other hand, this is less surprising if the stress-dependence of TTS is highlighted. A study of the mechanics of inelastic buckling reveals that, when the column starts to bend, the bending causes differential stresses between the concave and convex sides. At elevated temperatures this difference of compressive stresses will cause differential TTS, which further increases the differential total strain between the two sides, leading to further bending. Due to the rather large magnitude of TTS this interactive effect will be significant.

Fig. 6 also shows that the result of the analysis without TTS lies between the two theoretical buckling loads (the dashed and chain-dot curves), which is consistent with Shanley's theory. The results of the transient heating analysis are shown as the black triangles and diamonds plotting the ultimate buckling temperatures of the model at various load levels, with and without considering transient thermal strain. Their comparison with the results of the steady-state heating analysis indicates little difference between these two heating scenarios, suggesting that the steady-state heating approach is sufficient, at least when the cross-section is uniformly heated, although the influence of the heating scenario is expected to be more significant if a thermal gradient exists through the cross-section. In particular, the magnitude of the additional stresses and strains caused by the thermal gradients should be significantly affected by the stress history.

2.3 Comparisons of high-temperature concrete material models involving TTS

Apart from Anderberg's model, two other mathematical models of the material properties of concrete at high temperature, which involve the transient straining property, have also been applied to the numerical analysis. These two models, which are given by Schneider [6] and Khoury [2] respectively, are briefly noted below.

2.3.1 Schneider's model

In this model, the transient strain ε_{tr} and the basic creep ε_{cr} are combined, and thus there are only three strain components:

$$\varepsilon = \varepsilon_{th} + \varepsilon_{\sigma} + \varepsilon_{tr,cr} \tag{3}$$

$$\varepsilon_{\sigma} = \frac{1+\beta(T,\sigma)}{g(T,\sigma)} \cdot \frac{\sigma}{E} \tag{4}$$

$$\varepsilon_{tr,cr} = \frac{\Phi(T,\sigma)}{g(T,\sigma)} \cdot \frac{\sigma}{E} \tag{5}$$

where β, g and Φ are empirical functions of stress and temperature, and E is the initial tangent modulus of the stress-strain curve at temperature T. The function β is to account for the increase of plasticity of the material as stress level increases, and g takes account of the increase of the initial tangent modulus of the stress-strain curve when pre-stress applies.

2.3.2 Khoury's model

Similarly to Schneider's model, the total strain consists of three components in this model:

$$\varepsilon = \varepsilon_{th} + \varepsilon_{\sigma} + \varepsilon_{tr,cr} \tag{6}$$

$$\varepsilon_{\sigma} = \sigma / E_0 \tag{7}$$

where E_0 is the initial tangent modulus of the concrete material at room temperature.

Distinguishing it from the previous two models, in this model ε_{σ} covers only the linear-elastic range of the instantaneous stress-induced strain derived from the stress-strain curve, ignoring the change of initial tangent-modulus as temperature rises; thus it is called elastic strain. The plastic instantaneous stress-induced strain is combined with the transient thermal strain and the basic creep strain as a single strain component called load-induced thermal strain (LITS). For a stress level equal to $0.3\sigma_{u0}$, LITS is an empirical function of temperature $g(T)$,

$$LITS(T, 0.3\sigma_{u0}) = g(T) \tag{8}$$

For other stress levels, the equation is modified to

$$LITS(T,\sigma) = LITS(T, 0.3\sigma_{u0}) \times \left(0.032 + 3.226\frac{\sigma}{\sigma_{u0}}\right) \tag{9}$$

where σ_{u0} is the ultimate stress at ambient temperature.

2.3.3 Model comparison

Schneider's and Khoury's models have been implemented into the Shanley-like analysis and compared with Anderberg's model. Since the decomposition of the total strain varies in these three models, it is not possible to compare each strain component from different models separately, and therefore the comparisons shown in this section are all based on the total strain including the transient straining property.

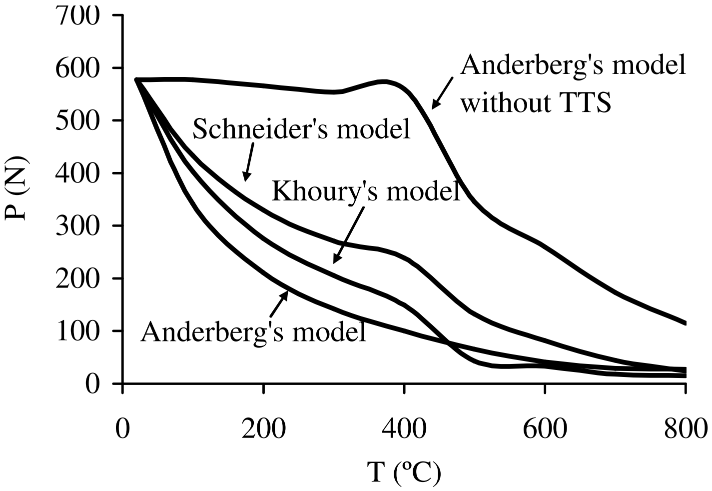


Fig. 7 – Buckling resistance of the multi-spring strut model using various high-temperature concrete material models involving TTS.

Figure 7 shows the ultimate buckling loads of the multi-spring strut over a range of temperatures, applying these three high-temperature concrete material models. It is seen that the three curves are reasonably close to each other at either low or high temperatures whist they split evidently from each other in the temperature range from 200 ºC to 400ºC. Anderberg’s model leads to the most conservative prediction of the buckling resistance, except that in the region of 550ºC Khoury’s model gives the lowest buckling loads. Although the strain definitions and formulations of the models are completely different from each other, they all show a reduction of the buckling resistance caused by TTS or LITS, compared with the case in which Anderberg’s model is applied without the TTS component.

3. FE ANALYSIS WITH *VULCAN*

The analysis on the simplified Shanley-like model was followed by complementary finite element analyses. The FE software *Vulcan* [11], which has been specially developed for structural fire engineering analysis, was further developed to take into account the transient thermal straining property of concrete. The 3-noded beam element of *Vulcan* [11], whose general cross-section consists of a finite number of segments, proved suitable for such development. Each segment is assumed to be fully in contact with adjacent segments, and their relative movements are restricted by the assumption that plane cross-sections remain plane. If the Poisson's ratio of the material is set to zero, these segments should stay reasonably uniaxial, and effectively replace the springs in the Shanley-like model, whilst taking into account the material continuity along the length of the column.

As with the Shanley-like-model, the temperature distribution through the cross-section was assumed to be uniform at this stage, and Anderberg’s transient thermal strain model was implemented. A three-metre long, 200mm square, bare concrete column, simply-supported and subject to pure compression under transient heating, was examined. It should be noted that, for a suitable example, the dimensions and material of the column must ensure that the column fails by buckling in the inelastic range, in which case, the tangent-modulus critical load F_t must be smaller than the compressive strength F_u and larger than the proportional limit F_p of the column. The analysis reveals overall buckling as the major failure mode of the column, and the effect of transient thermal strain on this buckling is illustrated in Fig. 8. This figure shows the failure temperatures of the column at various load levels, both with and without considering transient thermal strain. TTS causes reduction of the buckling resistance of this column as it does to the Shanley-like model. The two curves without TTS in Figures 6 and 8 have different shapes, because different stress-strain curves were used in these two analyses; Anderberg's curve for the former and the EC2 curve for the latter.

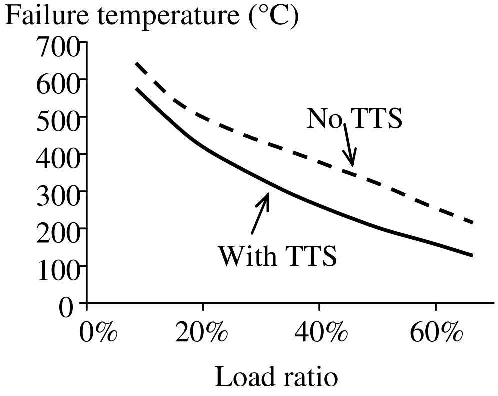
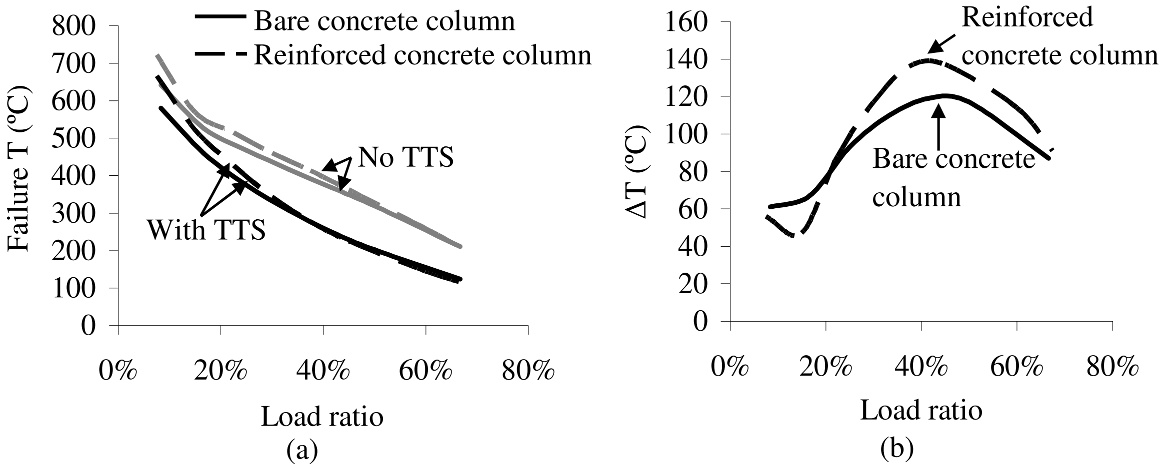


Fig. 8 – Effect of transient thermal strain on the overall buckling of the 3m plain concrete column from *Vulcan* analysis.

This investigation was then extended to reinforced concrete columns. Maintaining the dimensions of the cross-section and the length of the column as for the bare concrete column above, four rebars of 10mm diameter with 25mm cover from the column surface were added. The failure mode of the column is still overall buckling. Fig. 9(a) plots the failure temperatures of this column with or without TTS at various load levels (the dashed curves) which are compared with those of the bare concrete column (the solid curves). The results of the analyses with and without TTS are distinguished by line colours as marked. It is shown that the introduction of TTS causes a reduction of the buckling resistance of the RC column which is similar to that of the bare concrete column.

The decrease of the failure temperature caused by TTS at various load levels for these two types of columns is illustrated in Fig. 9(b). For both types of column, the reduction of the buckling resistance due to TTS is most significant at the intermediate load level when the load ratio (the ratio of the applied load to the cold strength of the column) is around 40%. This may be because the magnitude of TTS depends on the combination of temperature and stress level as implied by Equation (2), and it turns out to be largest when both of these two factors are in the intermediate range. Fig. 9(b) also indicates that, except when the applied load is very low, the reduction of the buckling resistance caused by TTS is more significant for the RC column rather than for the bare concrete column. As demonstrated by the Shanley-like model, such a reduction is very directly related to the amount of lateral deflection occurring, and it is clear that the RC columns have higher bending stiffness than their bare concrete counterparts. This may explain the reason that TTS causes the greater reduction of the buckling resistance to the RC column.



(a) (b)

Fig. 9 – Comparison of the effect of transient thermal strain on the overall buckling of the reinforced and bare concrete columns from *Vulcan*.

The effects of the steel casing of concrete-filled columns on the way TTS affects the buckling of such columns at high temperature were also investigated. Two three-metre long concrete-filled columns, made of 10mm and 15mm thick, square steel tubes with a 200 mm square concrete infill, were examined. The effects of TTS on the overall buckling of these columns and bare concrete columns of the same size as their concrete cores were compared, and the results are shown in Fig. 10. TTS still causes a reduction of the buckling resistance of the concrete-filled columns, but this reduction is much smaller than for the concrete and reinforced concrete columns examined previously. It is also shown in Fig. 10(b) that the reduction decreases with increasing thickness of the steel casing. This indicates, logically, that as the thickness-to-width ratio of a concrete-filled column increases, the behaviour of the column is more influenced by the behaviour of the steel tube, and therefore the effect of the transient strain of the concrete infill may not be as significant as for concrete and reinforced concrete columns. However, since the temperature of the concrete core has been overestimated by the simplified assumption that the column is uniformly heated through its cross-section, whilst the effect of TTS is expected to be more significant when a more realistic temperature profile, in which the concrete core is cooler, is applied. Investigation of the effect of the thermal gradient through the cross-section is on-going.

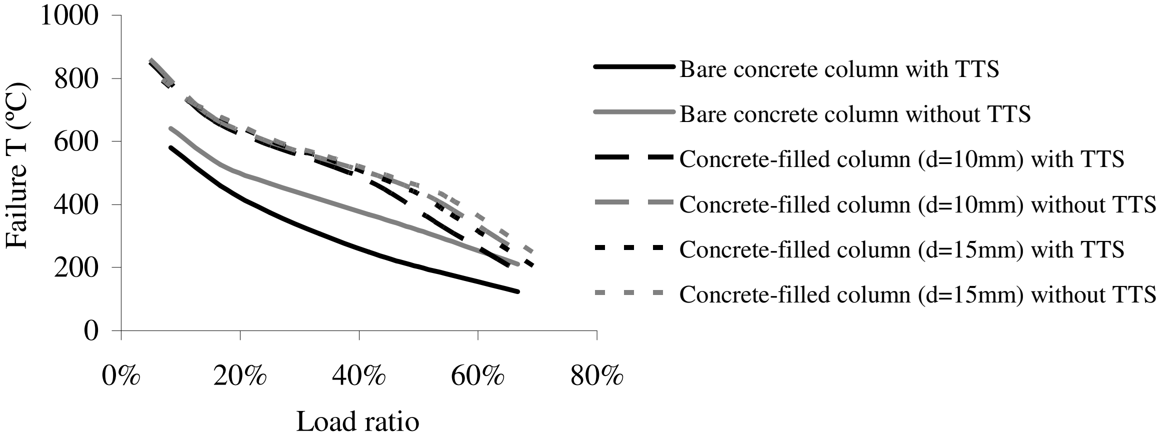


Fig. 10(a) – Comparison of the effect of transient thermal strain on the overall buckling of the concrete-filled and bare concrete columns from *Vulcan*.

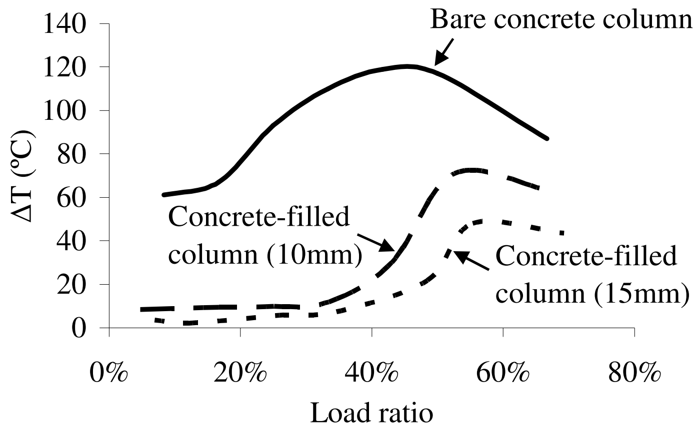


Fig. 10(b) – Comparison of the effect of transient thermal strain on the overall buckling of the reinforced and bare concrete columns from *Vulcan*.

4. CONCLUSIONS & FUTURE WORK

The classic simplified model due to F.R. Shanley, of a column buckling in the material's inelastic range, has been extended and its characteristics programmed in an initial study of the mechanics of inelastic buckling. High-temperature analysis has also been conducted on this model and three high-temperature concrete material models involving TTS have been compared. This has been followed by complementary FE analyses, developing the software *Vulcan*. Both analyses show a remarkable reduction of the buckling resistance of uniformly heated concrete columns caused by TTS, revealing the potential dangers of neglecting this property of concrete in structural analysis and design for fire. Preliminary parametric studies on both concrete and concrete-filled columns concerning the effects of reinforcement and steel casing have been carried out, indicating that the reduction of the buckling resistance caused by TTS is more significant for uniformly heated RC columns compared to bare concrete columns of the same size, and that as the thickness of the steel casing of the concrete-filled columns increases, this reduction decreases.

The effects of thermal gradients through the cross-sections of columns due to rapid heating, both symmetric and asymmetric, are being considered. The FE analyses using *Vulcan* need to be further validated against the Shanley-like model. The analysis will then be compared with test results and current design methods. The investigation will later move on to the effect of TTS on columns in continuous structures rather than single members.

Acknowledgment: *The principal author is grateful for the support of Corus Group Ltd and the Engineering and Physical Sciences Research Council of the United Kingdom, under a Dorothy Hodgkin Scholarship.*

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