A UNIFIED RHEOLOGICAL MODEL FOR ANALYSIS OF STEEL BEHAVIOUR AT HIGH TEMPERATURE

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ABSTRACT

This paper presents the theoretical background for development of a unified rheological model (URM) for low-carbon steel which is sensitive to the temperature and strain-rate effects during fire exposure. The proposed URM consists of a serial connection of two Kelvin-Voight (KV) rheological elements. Each KV element is used to simulate a particular material strain (stress-related and creep strain) in fire-affected steel. The first KV element is capable of taking into account strain-rate-governed change of yield strength and the second is sensitive to variation of heating rate. A verification of the proposed URM by using test data from transient coupon tests of steel Grade 275 is also presented.

INTRODUCTION

A traditional representation of a material model for fire-affected steel [1-3] is based either on an analytical description of the test results from a set of stationary or transient coupon tests conducted at high temperature. These coupon tests are generally conducted by using a prescribed strain or stress rate at given temperatures in a stationary test, or by using a prescribed heating rate at constant stress in a transient test. In both cases, the material model obtained only captures a limited range of material effects (and behaviour patterns) which occur during high-temperature exposure, depending on the test parameters used.

An example of a material model based on a fixed heating rate, which is widely used in Europe, is the Eurocode 3 material model [1]. This model was determined on the basis of series of transient tests conducted at 10°C/min [4-6]. The test parameter used for his particular model left an open question about its validity in accounting for steel creep which occurs below the test heating rate.

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Some recent experimental/numerical studies [7-9] have pointed out that the Eurocode 3 material model has limitations in modelling creep in the case of heating rates lower than 10°C/min. These studies have suggested that an explicit-creep analysis should be incorporated when analyzing fire response in the case of low heating rates. Considering the traditional testing procedures for determining steel's mechanical properties, two distinct test parameters can be identified.

The first is the heating-rate, which is generally considered as a factor that governs the level of creep strain evolution in fire-affected steel [7-9]. The second is the strain-rate, which is considered [10] as influencing the yield strength of steel, which tends to increase with the increase of strain-rate. Therefore, a universal material model which is sensitive to these crucial parameters cannot be created if relying solely on either of the traditional test procedures used for determining material properties. A unified rheological model, such as the one presented in this paper, represents a way of taking into account the complex response of the material and its dependence on the crucial heating and loading boundary conditions.

THEORETICAL BACKGROUND

Three strain components for steel at any temperature can be defined according to [11]:

$$\varepsilon_{\mathrm{tot}} = \varepsilon_{\mathrm{th}}(T) + \varepsilon_{\sigma}(\sigma, T) + \varepsilon_{\mathrm{cr}}(\sigma, T, t) \tag{1}$$

in which: $\varepsilon_{\mathrm{tot}}$ is the total strain, $\varepsilon_{\mathrm{th}}(T)$ is the thermal strain, $\varepsilon_{\sigma}(\sigma, T)$ is the stress-related strain and $\varepsilon_{\mathrm{cr}}(\sigma, T, t)$ is the creep strain. As is generally appreciated, thermal strain is temperature-dependent, stress-related strain depends on the applied stress σ and temperature T, and creep strain depends on stress, temperature and time. The stress-related and the creep strains are the most complex to define, due to their dependency on a large number of thermo-mechanical variables. The thermal strain will not be considered in the following representation of the rheological model since it can be modelled with relative ease. The proposed URM consists of a series connection of two Kelvin-Voight (KV) rheological elements. Each KV element is used to simulate a part of the material strain.

The first KV element represents a stress-related strain component which defines the strain-rate governed change of yield strength. The second KV element represents a creep strain component which is sensitive to variation of heating rate. The differential equation describing the strain calculation procedure for a series connection of KV elements is:

$$\frac{\sigma}{\mathrm{c}_{\mathrm{i}}} = \frac{\mathrm{k}_{\mathrm{i}}}{\mathrm{c}_{\mathrm{i}}} \varepsilon_{\mathrm{i}} + \dot{\varepsilon}_{\mathrm{i}} \,;\, \mathrm{i} = 1, 2 \quad ; \quad \dot{\varepsilon} = \mathrm{const} = \dot{\varepsilon}_1 + \dot{\varepsilon}_2 \quad \mathit{or} \quad \sigma = \sigma_{el1} = \sigma_{el2} \tag{2}$$

In which $\dot{\varepsilon}_{\mathrm{i}}$ represents the first strain derivative of the i-th KV, and k_{i} and c_{i} represent the spring and damper functions. These nonlinear functions are stress-, temperature- and strain-rate dependent, so they which can be written as:

$$\sigma_1 = k_i(\sigma, T)\varepsilon \quad ; \quad \sigma_2 = c_i\left(\dot{\varepsilon}, T\right)\dot{\varepsilon} \quad ; \quad i = 1,2 \tag{3}$$

Figure 1 shows the rheological model and its constitutive components.

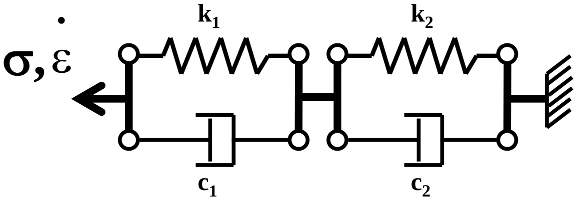

Figure 1. Unified rheological model and its constitutive components.

The solution of two differential equations (2) can be obtained using Euler integration with the appropriate increment for time integration. Two types of solver (strain-rate or stress-rate) can be employed, depending on the type of test which is being analyzed. The principle of a stress-rate solver is illustrated in Figure 2. This type of solver is appropriate for modelling transient tests, in which stress is a fixed test parameter. A different type of solution scheme has to be employed for modelling strain-rate-controlled tests.

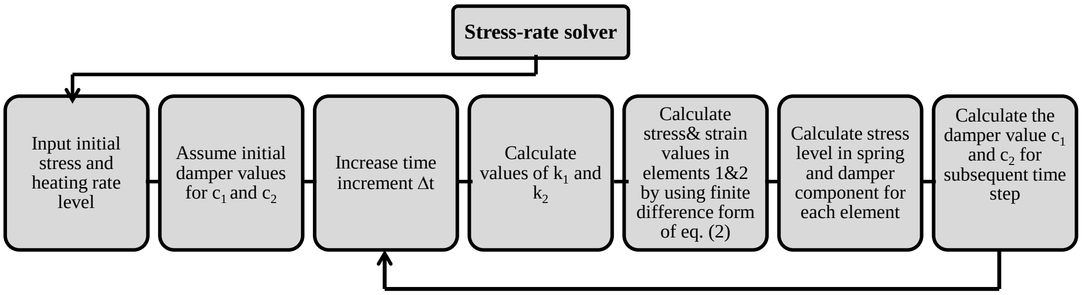

Figure 2. Principle of a stress-rate solver.

COMPONENT MODELS

Prior to the utilization of the unified rheological model, constitutive material laws need to be provided for all four components of the URM. The spring component model is based on a previously developed [8, 12] creep-free Eurocode 3 stress-strain model. Essentially, its shape and analytical form is identical to the original Eurocode 3 model, except that the yield strain value has been modified, and in the creep-free version the yield strain amounts to 1% instead of the original 2%. This is the constitutive model for the spring component belonging to the first KV. The constitutive model for the second spring has the same shape and form as the model for the first, except for the modified value of yield strength (f_y) which amounts to 80% of that used for the first spring. This was arranged in order to take into account the experimentally observed reduction in yield strength at very low strain-rate high-temperature tests [10]. The shapes of the creep-free stress-strain models for both springs are presented in Figure 3.

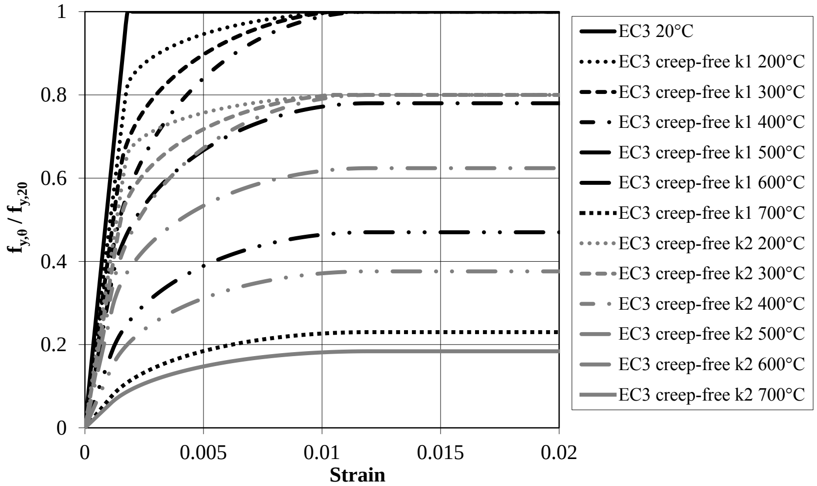

Figure 3. Graphical representation of constitutive material models for spring 1&2

The constitutive model for the damper component of the first KV has been determined with the help of strain-rate test results from study [10]. The constitutive model for the second damper was created with the help of an existing creep model developed by Harmathy [13] and the corresponding material parameters for American steel A36 [14]. This creep model has proved sufficiently accurate in representing the creep-strain behaviour of European steel grades, which was analyzed in previous research studies [7-9] by the authors. The damper value for both KVs is calculated by the URM using the following expression:

$$c_i = \frac{\sigma_{d_i}}{\dot{\varepsilon}_i}\,;\; i = 1,2 \tag{4}$$

where σ_{d_i} is the damper stress and $\dot{\varepsilon}_i$ is the strain rate of the corresponding KV element. In the case of damper c_1 of the first KV, the damper stress is determined from an experimentally-derived function, and the strain rate can be determined after the calculation of strain of the first KV is completed. The damper stress of the first KV is a function of both strain rate and temperature. This relationship has been determined experimentally, using data from study [10] for slow, medium and fast strain-rate tests. For the second damper, the damper stress is determined using the value of c_2 from the previous time step. The strain rate of the second KV (which in this case represents the creep strain rate) is determined as a function of temperature and stress (defined on a logarithmic scale). The values of c_1 and c_2 which are calculated in the current time step are stored for use in the subsequent time step.

MODEL VERIFICATION

The overall performance of the unified rheological model is compared with the test results published by Kirby *et al.* [4, 5], using the results of transient tests conducted at heating rate of 5°C/min for steel S275 and at 2.5°C/min for S355.

Selected results are given in Figures 4 and 5, in which a comparison is given between the predictions of the URM and the transient coupon study [4, 5]. The unique feature of the proposed URM is, as specified in Equation (3), that the spring and damper components are temperature-, stress- and strain-rate-dependent. Therefore, by providing adequate calibration of the constitutive models for springs and dampers, it is possible to take into account the effects of variable strain- and heating-rate on the strain output. The comparisons from Figures 4 and 5 show that the URM can adequately model the development of stress-related and creep strains by using the proposed material models for each of the constitutive components.

A discrepancy can be observed from Figure 4 at higher stress levels, which can be attributed to the fact that the yield strength increase has been taken into account up to a certain strain-rate level (the fastest strain-rate from study [10]). In cases where steel is stressed close to its yield strength, the occurrence of a very high strain rate is possible, especially at and beyond the steel's yield strain. This is why the URM shows some discrepancy in predicting strain evolution after the yield strain which is used in the URM (1% strain in Figure 3) is exceeded.

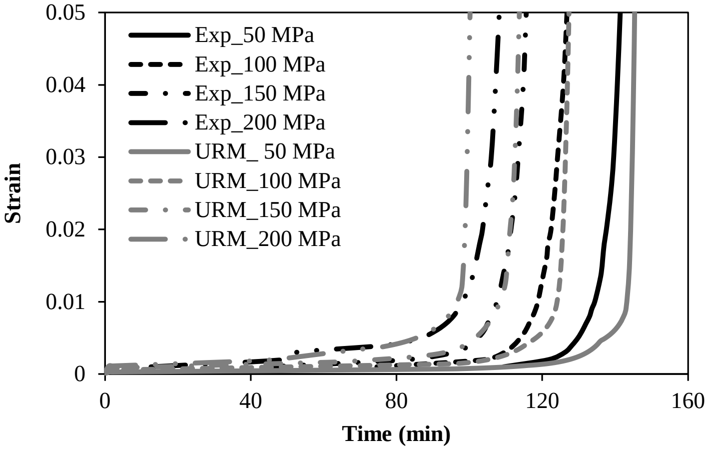

Figure 4. Selected comparison of results between the URM and the coupon experiments - grade S275 [4,5] – 5°C/min.

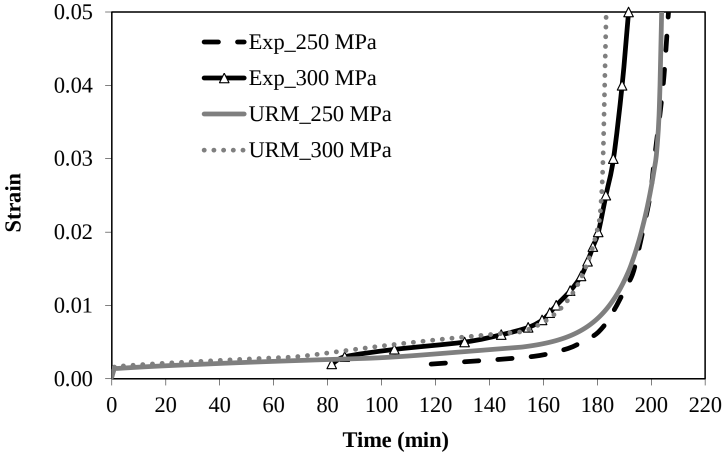

Figure 5. Selected comparison of results between the URM and the coupon experiments - grade S355 [4,5] – 2.5°C/min.

Figure 6 presents the values of damping constant c_2 calculated by the URM for transient testing at 5°C/min, where a coupon of steel S275 is exposed to a stress of 100 MPa. The main logic behind the interaction of two dampers as strain-rate-sensitive components is that the initial value of c_1 is very low in comparison to the value of c_2. This is to ensure the activation of damper c_1 at higher strain-rates, when the change of yield strength occurs, and the activation of damper c_2 at later stages of fire exposure when creep starts to develop. The functionality of the damper c_2 is illustrated in Figure 6, where the initial damping value is initially very high (no creep strain), and after 140 minutes a sudden drop in the value is observed (which corresponds to the start of creep strain development).

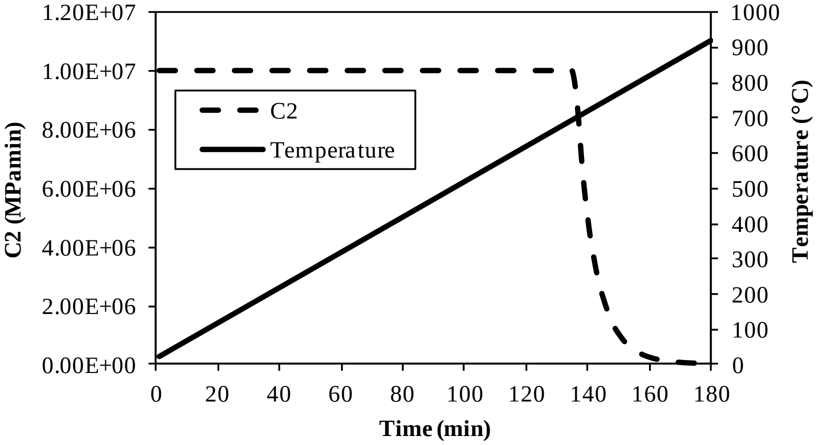

Figure 6. Reduction of the value of c_2 with respect to time – S275, 100 MPa at 5°C/min

A comparison between the explicit creep analysis using the original Eurocode 3 material model and the predictions of the URM is given in Figure 7. This analysis was conducted by utilizing the *Vulcan* research code and its force-controlled solver, which can run the analysis up to the onset of the yield strain of original Eurocode 3 model (2%).

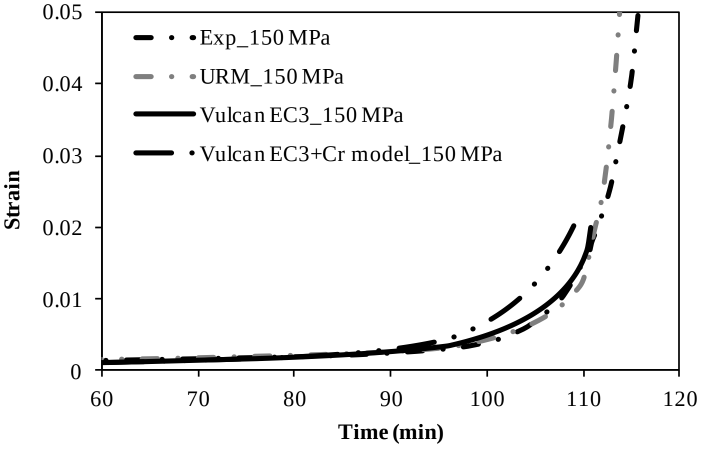

Figure 7. Comparison of different modelling approaches – URM vs. Vulcan explicit creep modelling for grade S275, 5°C/min

It can be seen from Figure 7 that the analysis which uses the original EC3 model in combination with an explicit creep model provides imprecise predictions of the total strain if compared to the experimental values, with the predictions of the URM being closest to the experimental values. In addition, the URM provides good predictions of total strain over a range of 1%, indicating its validity over the entire strain range.

Table 1. presents a comparison of results between the predictions obtained by the URM and the experimental values.

TABLE I. COMPARISON OF TOTAL STRAIN PREDICTION – URM AND THE EXPERIMENT

Temperature (°C)	**Exp [4,5] 5°C/min, S275 (%)**	**URM (%)**
558	1.00	0.9
563	1.20	1.04
567	1.40	1.19
572	1.60	1.60
574	1.80	1.78
577	2.00	2.13

CONCLUSION

Variation of both strain- and heating-rates is generally expected in fire-affected steel structures. Therefore, a versatile material model is necessary in order to take into account all the effects which occur in the structure when exposed to variable mechanical and thermal boundary conditions. The main advantage of the proposed URM is that it can take into account the change of crucial thermo-mechanical variables, since a classical material model based either on a test conducted with a unique strain- or heating-rate cannot be considered as representative for all aspects of overall material strain output during fire exposure. Future research will involve an in-depth verification of the URM by using published test studies [4, 5], including other available test sources, whether these are based on stationary or transient test data.

Acknowledgement

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