TENSILE MEMBRANE ACTION OF COMPOSITE SLABS IN FIRE - ARE THERE ANY RELIABLE DESIGN METHODS?

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Abstract
Following the Cardington fire tests in the mid-1990s tensile membrane action was identified as a key reason why composite slab panels with unprotected downstand steel beams did not fail structurally, although their temperatures reached values well in excess of codified limiting values. Initially in the United Kingdom, simplified analytical methods were published, based on yield-line theory but extended to large deflections, for the lightly-reinforced concrete slabs which form the continuous part of any composite floor panel. This paper presents a reappraisal of the most popular of these methods in the context of a more systematic approach based on the kinematics of the facets of a rectangular panel at finite deflection, after an initial yield-line mechanism has formed. The new approach allows for fracture of mesh across yield lines, which could be a real issue for the localised cracking, which is particularly relevant to lightly reinforced slabs, especially since current practice is to use deformed bars in the welded mesh, rather than the plain bars used in the Cardington building. For composite slabs it uses the yield-line pattern appropriate to the composite slab at high temperature as a basis, and then directly calculates the enhanced steel beam temperature at finite deflection. Again, the key behaviour in limiting the possible enhancement of beam temperature is the initiation of mesh fracture. Finally the effect of thermal bowing due to the temperature difference between the steel beams and the concrete slab is investigated. While thermal bowing increases the yield-line temperature, it does not appear to affect the enhancement due to subsequent deflection to a great extent.

Keywords: Tensile membrane action, Simplified methods, Composite slabs, Fire

1 INTRODUCTION

Tensile membrane action of composite floor slabs has been of interest in research and design since the full-scale fire tests conducted on a composite building at Cardington [1] during 1995-96. In these tests, in which the steel downstand beams were left unprotected against fire, no structural or integrity failures were observed, despite the slabs experiencing extremely large deflections. In these tests, the steel beams reached temperatures well in excess of the failure temperatures, based on standard fire testing of composite beams, which are included in current codified design [2]. Since the steel beam temperatures reached levels at which the concrete slabs were essentially non-composite, it was clear that their large double-curvature deformations had enhanced their load-carrying capacity very considerably. The mechanism generated by large double-curvature deflection, known as tensile membrane action (TMA), is one in which bending at small deflections is progressively replaced by biaxial membrane tension in the central area of a slab, equilibrated against a ring of membrane compression around the vertically-supported edges. This was an effect that had previously been researched [3, 4], with a series of papers being published [5, 6, 7, 8] during the 1960s. In particular, the work of Hayes [7, 8] had introduced a relatively simple method of calculating the enhanced plastic capacity of a concrete slab, based on the optimal yield-line pattern [9] generated at infinitesimal deflection and related to the slab’s finite deflection.

In the period following the Cardington tests, it became clear that the opportunity existed for designers to take advantage of the enhancement of slab capacity provided by TMA in compensating for the loss of strength of downstand steel beams at high temperatures, as an alternative to simply providing fire protection to all exposed steelwork of composite floors. The only pre-requisite to generating TMA in a rectangular slab panel is to provide adequate vertical support around its perimeter; apart from this, the mechanism is self-equilibrating. This led to the development of a fire resistance design strategy for composite floors (Fig. 1) in which slab panels bounded by primary and secondary composite beams connected directly to the column grid were to be treated as discontinuous across
these edges. The perimeter beams on the column grid should be protected in order to provide the required vertical support, but the internal secondary beams could be left unprotected.

For individual slab panels the key design calculations were to determine the deflection necessary to provide the required resistance, allowing for the loss of strength of the downstand beams, and the limiting deflection at which real structural failure would take place. The latter, in particular, would require an appropriate safety margin.

Fig. 1: (a) Protection strategy to utilise TMA; (b) Individual panel model.

2. HAYES’S METHOD AND ITS ADAPTATION

The work of Hayes forms the basis of the main TMA models developed after Cardington. Hayes’s premise, for purely reinforced concrete slabs at ambient temperature, was that the classic optimal yield-line pattern of linear plastic hinges forms at infinitesimal deflection, at the critical yield-line failure load. As the load is increased, this pattern remains fixed, but the deflection needs to increase in order for TMA to sustain the increased load. Hayes assumes that the yield-lines then carry plastic moments from the reinforcing mesh in orthogonal directions, but in addition to these the yield lines are subject to linear distributions of membrane force and net shear forces, as shown in Fig. 2.

Fig. 2: Hayes’s membrane force assumption. (a) Rationale for membrane distribution; (b) Equilibrium with short-span tension cracking.
Another aspect of importance is that in tests [10] of slabs in TMA, one or more pure tensile cracks through the slab have been observed at a late stage, across the short span of the slab. In some cases these tension cracks have originated at the intersection of the yield lines, but in the majority there has been a single transverse tension crack located at the middle of the long span. In Hayes’s analysis, the tension crack has been assumed to pass through the yield line intersection, as shown in Fig. 2(b). The membrane force distributions are assumed to be fixed, and do not change with deflection of the slab. From equilibrium in the horizontal plane (Fig. 2(b)) the extreme values \( bKT_0 \) and \( kbKT_0 \) are calculated. The presence of either tensile or compressive membrane force at any point along a yield line reduces the plastic moment capacity at this point, and this is also calculated. Since the yield-line pattern is bisymmetric, it comprises two of each type of flat facet; triangular and trapezoidal. Enhancement factors of capacity due to membrane forces \( e_{1m} \) and \( e_{2m} \) are calculated independently for each of these facets by taking moments of the deflected membrane force resultants about their supported edges. Since the plastic yield-line moments have been reduced by the presence of membrane force, the net effect of this on the yield line capacity is taken into account as a separate “enhancement” factor (actually a reduction factor) for each facet as \( e_{1b} \) and \( e_{2b} \). The enhancement factors for the individual facets are added, to form two net enhancements \( e_1 = e_{1m} + e_{1b} \) and \( e_2 = e_{2m} + e_{2b} \) which are generally unequal. These are compounded into a single enhancement factor using the equation:

\[
e = e_1 - (e_1 - e_2) / (1 + 2\mu\alpha^2) \tag{1}
\]

Where \( \alpha \) is the aspect ratio of the slab and \( \mu \) is the mesh orthotropy ratio. The equation is not explained. In unpublished work Gillies [11] has rationalised this by including the vertical shear resultants on the diagonal yield lines, to:

\[
e = e_1 - (e_1 - e_2) / (1 + 2\mu\alpha^2) \tag{2}
\]

In Hayes’s work, Equation (1) gives the enhancement of yield-line capacity due to deflection. This is linear with deflection, and depends also on the aspect ratio \( \alpha \) of the slab; the nearer its value is to unity, the greater the enhancement. At zero deflection, the value of the enhancement factor is always lower than unity, and so enhancement does not happen below a certain value of deflection. The concept of a deflection at which real structural failure takes place does not exist in Hayes’s analysis; it does not consider mesh fracture.

In the adaptations [12, 13, 14] of Hayes’s method for use with composite slab panels in fire, the approach outlined above has been adopted with just one change. The pure tension crack across the short span of the slab is located at the centre of the long span. The equilibrium of forces that determines the values of \( bKT_0 \) and \( kbKT_0 \) is based on the system of forces shown in Fig. 3. The factor 1.1 on the tension crack force represents ultimate rather than yield conditions.

![Fig. 3: Equilibrium at short-span tension cracking for fire methods.](image-url)
Since this enhancement applies only to the capacity of the concrete slab, taking no account of the fact that it acts compositely with downstand steel beams, the reduced capacity (in kN/m²) of an unprotected composite beam when the steel downstand is at its maximum temperature is added to the enhanced capacity of the slab at the corresponding deflection. It is necessary to estimate this limiting deflection, to avoid mesh fracture, in order to avoid a catastrophic structural failure which would allow fire to spread beyond the compartment of origin.

A simple method, which includes safety factors, has been included to limit the permissible deflection range and permit calculation of the maximum feasible enhancement of yield-line capacity. The limiting deflection is calculated using simple models of thermal bowing of the concrete slab and the strain in reinforcement caused by deflection. These two deflections are simply superposed as illustrated in Fig. 4.

The thermal bowing deflection (Fig. 4(a)) is calculated for a slab-strip aligned in the short-span direction; in the original method [12, 13] this used a linear temperature gradient through the depth of the slab; the temperature difference was 770°C, which simply reproduced measurements taken at Cardington. In software generated to support further development a one-dimensional thermal analysis is used instead.

![Fig. 4: Deflections: (a) due to tensile straining over a fixed span; (b) due to thermal bowing over a simply supported span.](image)

However, the key fact about this displacement is that the model of the slab-strip must be simply supported, in order to allow the thermal bowing to occur. The mechanical deflection, which is added to this, is calculated on the basis that a long-span slab strip undergoes a uniform tensile strain of half of the ambient-temperature yield strain of the steel longitudinal reinforcement, and that this addition to the span is accommodated in a parabolic deflected shape, which fits the original long-span dimension. This requires the slab strip to have fixed boundary conditions. Although the two deflection components are calculated for orthogonal slab strips, they cannot legitimately be superposed.

Using the limiting deflection given by this superposition the maximum enhancement to the concrete slab capacity can then be calculated. The maximum temperature experienced by the unprotected downstand steel beams is known, either from the desired fire resistance time or from a worst-case parametric fire scenario, and this can be used to calculate the reduced floor load capacity of each of the composite beams that together make up the slab panel. This reduced capacity as an array of composite beams is added to the enhanced capacity of the concrete slab at its limiting deflection.

In summary, the method is based on the effect of finite deflection on a classic yield-line pattern of plastic hinges in a concrete slab. For very lightly reinforced thin concrete slabs, such as the usual
type to which the method refers, the yield lines would be localised rather than spread across wider zones, which makes this approach legitimate. However, there are several arguments against its validity:

- The yield line pattern assumed is that for a concrete slab rather than a composite slab. Even when the downstand beams are at high temperature, they effectively constitute extra reinforcement in their own direction, and therefore affect the actual yield-line pattern.
- The assumed membrane force distribution is independent of deflection. It does not allow for the possibility of through-depth cracks along the yield lines.
- Reinforcement is assumed infinitely ductile across the yield lines, and is not assumed capable of fracturing.
- The enhancement factor for TMA is below 1.0 at zero deflection.
- Addition of the capacity of the non-composite slab in TMA to the capacity of the heated composite beams uses the concrete slab twice, assuming different stress situations.
- The superposition of deflections based on thermal bowing and mechanical strains to form a limiting deflection is unjustifiable.

3. REAPPRAISAL OF FINITE-DEFLECTION YIELD-LINE BEHAVIOUR OF A COMPOSITE SLAB

Within the principles of yield-line analysis, the author has conducted a reappraisal [15, 16] of the behaviour of a set of slab facets, whose shapes are determined by the small-deflection optimisation and do not subsequently change. The fundamental principles of the analysis are:

It obeys the kinematics of the facets rotating about their supported edges, causing the facets to overlap in areas where the concrete is in contact across a yield line, as shown in Fig. 5. The same deflection applies to each facet.

Fig. 5: Geometry of diagonal yield-line crack opening. (a) Crack opening at rebar level; (b) Top surface of slab, including rigid-body movements.

- In areas of contact of concrete across yield lines the concrete acts at constant stress.
- Where intact reinforcing bars in either alignment cross a gap between concrete surfaces along a yield line they act at their constant yield strength.
- When the concrete surfaces along a yield line in either alignment reach a certain separation (the fracture crack-width) the reinforcement in that alignment fractures. Reinforcement in zones of concrete contact is not included.
- The forces acting on each of the facets are in horizontal equilibrium.

At any deflection, the application of horizontal equilibrium, subject to the constraints of kinematics, dictates the state of concrete contact and reinforcement fracture. The load capacity is then calculated.
by taking moments about the supported edges of each of the two distinct types of facet and eliminating the net vertical shear forces between the two equations produced. A sequence of elevations on the central and diagonal yield line cracks as the deflection increases from the pure yield-line bending failure at zero deflection is shown in Fig. 6. The concrete neutral axis can be seen to move upwards progressively at the yield line intersection and downwards at the slab corner in (a) until it becomes triangular in (b), beyond which the concrete has lost contact completely along the central yield line. At some point the reinforcement crossing the central yield line fractures, and subsequently the reinforcement in the two alignments fractures (“unzips”) along the diagonal yield lines, from the intersection towards the corners. It is possible finally to create a trapezoidal concrete stress block, as shown in (d), although in practice this is very unusual.

![Diagram of yield lines and stress blocks](image)

**Fig. 6:** Projection on the x-direction of the yield lines at different stages; (a) Concrete stress blocks on all yield lines; (b) Triangular stress blocks above rebar on diagonal yield lines; (c) Triangular stress blocks below rebar on diagonal yield lines; (d) Trapezoidal stress blocks on diagonal yield lines.

**Mid-slab yield line**
**Diagonal yield line**

4. **COMPARISON OF APPROACHES**

A useful context to compare the existing BRE/Fracof methods with the new approach is the test [13] carried out at BRE Garston in 1999. This was an ambient-temperature test on a 9500mm x 6500mm slab that reproduced a typical corner panel from the Cardington test building in its details, but omitted the central longitudinal downstand secondary beam in order to represent the high-temperature condition when this beam had lost most of its strength. The test was carried out as part of the validation process for the BRE method.

Fig. 7(a) compares the test results with the enhancement curve given by the BRE method and the enhancements given by the current method for two different mesh ductility values.
Fig. 7: Comparisons between the Fracof/BRE and current methods: (a) for Garston test panel (includes the test results); (b) for a square 6500mm x 6500mm panel.

The mesh used in the test was welded A142 consisting of undeformed steel bars which were tested and shown to have 12% fracture ductility. Since plain steel bars cannot be guaranteed significant bond with the concrete the gauge length over which this acts at fracture is the 200mm separation between welds to orthogonal bars. For deformed bars the ductility would be much lower; the 4% ductility curves plotted relate to the same gauge length. It can be seen that the enhancement gradient given by the current method corresponds closely to that given by Fracof/BRE, although the deflection at first bar fracture (along the whole central yield line) corresponds closely to that at which the test had to be terminated. The curves shown in Fig. 7(b) for a square slab illustrate that the apparent correspondence between the current method and Fracof/BRE for the Garston test slab is coincidental; Fracof/BRE predicts slab capacities nearly 20% higher, and with no peak capacity. For higher aspect ratios Fracof/BRE predicts progressively lower enhancements than the current approach.

4.1. New methodology including the central tension crack

The current method in the comparison shown in Fig. 7 assumes that the through-depth tension crack observed in tests has not formed. If this crack has happened across the centre of the slab the kinematics of the slab facets change. The stress blocks along the yield lines are now shown in Fig. 8.

Fig. 8: Failure mode including the central tension crack. Change of concrete stress blocks and mesh fracture on yield lines and the tension crack as deflections increase.
This mechanism includes an in-plane rotation and an inward movement of the trapezoidal facet, which create a rectangular compression stress block at the edges of the through-depth crack across the central short span. Unzipping of the reinforcing mesh is now initiated from the intersection of the yield lines or in the tension crack, starting at its intersection with the central yield line.

On Fig. 9 the enhancements shown in Fig. 7(a) are compared with those from the current approach when the central short-span tension crack is assumed to have happened. This reduces the enhancement, which is now considerably below the Fracof/BRE prediction.

![Graph showing comparison between Fracof/BRE and current methods](image)

**Fig. 9:** Comparisons between the Fracof/BRE and current methods (with and without the central tension crack) for the Garston test panel.

### 4.2. Composite slabs in fire

Composite slabs are designed as a parallel array of composite beams against ambient-temperature Ultimate Limit State design loadings. In the Fire Limit State, the design load intensity is considerably lower, but as the temperatures of unprotected steel downstand beams rise the strength of individual composite beams is reduced progressively. Even at these reduced strengths the presence of downstand beams aligned in just one of the edge directions gives the slab a very considerable degree of orthotropy, and this changes the yield-line patterns from the optimum for a concrete slab of the same dimensions.

![Diagram of forces in horizontal equilibrium](image)

**Fig. 10:** Forces involved in horizontal equilibrium of facets in a high-temperature yield line mechanism of a composite slab.

For a composite slab in fire, the load intensity is fixed, but the temperatures of the unprotected downstand steel beams are increased progressively. These beams lose their strength with rising temperature, and at a critical beam temperature a yield line mechanism forms. The associated yield line pattern is highly affected by the degree of orthotropy at the yield line temperature, and for the majority of practical cases the alignment of the central yield line is perpendicular to the downstand beams. Only for very lightly loaded panels (for which beam temperatures are extremely high) does
This alignment remain in the long-span direction, as would happen for the purely concrete slab. The yield line critical temperature can then be enhanced as the slab deflection increases, although mesh can fracture either abruptly or progressively at various stages. An example of the critical beam temperature changes that can occur as a slab deflects is shown in Fig. 11. The example is for a square 9m x 9m slab with two unprotected downstand beams; the panel reproduces an arrangement used in internal panels in the Cardington composite test building.

![Graph showing temperature enhancements](image)

**Fig. 11:** Downstand beam critical temperature enhancements with displacement for 9m x 9m panels with different ductilities. Slab load intensities are marked on individual curves. The temperature enhancements shown in Fig. 11 can be seen to depend critically on the ductility of the mesh across yield lines, defined in terms of the crack-width at the reinforcement level at which the bars fracture. This is clearly highly dependent on the level of bond between the bars and concrete. For 1mm fracture crack-width, which is of the order that might be expected for deformed bars of Ductility Class B, there is always a distinct limit to the enhancement of unprotected steel temperature that can be expected, at a deflection of the order of the effective depth. With just 5mm fracture crack-width a much greater enhancement is possible. This principle, that lower degrees of bond between reinforcement and concrete is advantageous to the fire resistance of composite slabs, goes against the normal logic of reinforced concrete design for ambient temperature. However, the reasoning is clear from the annotated graph of temperature enhancement against deflection shown in Fig. 12.

![Graph showing temperature enhancements](image)

**Fig. 12:** Enhancement sequence for 9m x 6m composite slab with central downstand beam, for different fracture crack-widths (1mm, 1.5mm, 2mm, 2.5mm, 3mm).
The basic curves, plotted with solid lines, show the temperature enhancement progress for a 9m x 6m with the same detailing as the square slab of Fig. 11, but with a single central 9m unprotected downstand beam; the load intensity is 3kN/m2. The basic curves are for a fracture crack-width of 1mm. The key events occurring as the deflection increases are marked, and it can be seen that the peaks of enhancement correspond to new mesh fracturing events. Beyond the point at which bars crossing the diagonal yield lines begin to fracture the enhancement falls continuously with deflection. The succession of dashed curves show the effect of increased fracture crack-width, which is particularly dramatic for this case.

4.3. The effect of thermal bowing on enhancement

Composite slabs with unprotected steel downstand beams are subject to thermal deflection due to differential thermal expansion before any form of “failure” has taken place. The steel beams heat very quickly as the fire temperature increases. The concrete slab is much more massive, and has much lower thermal conductivity than the steel beams, and develops a thermal distribution in which the fire-exposed surface is relatively hot, but this temperature falls very rapidly with distance away from this surface, remaining relatively cool at its top surface. Particularly when heated by the ISO834 Standard atmosphere curve, which rises rapidly in its early stages, the differential between the temperature of unprotected beams and the average slab temperature is very high. If it is assumed that the concrete remains at ambient temperature, and that there is perfect shear connection between beam and slab, it is possible to use classical Rayleigh-Ritz analysis to find the deflected shape of the slab at any beam temperature as the amplitudes of a double-Fourier series of half-sine deflection modes. The first Fourier mode is a very accurate approximation to the thermal deflection of the slab at any beam temperature. Since the optimum yield line pattern at any beam temperature is known, this can be matched (Fig. 13) with the first component of the Fourier series representation of the deflected yield-line mechanism, giving the equivalent thermal deflection of the central yield line.

![Graph](image)

Fig. 13: Representation of elastic thermal bowing of the slab as equivalent amplitude of the optimum yield-line mechanism.
An example of the effect of thermal bowing on enhancement of the beam critical temperature is shown in Fig. 14.

Fig. 14: Beam temperature enhancements for 9m x 6m composite slab with 8 kN/m² loading, both with thermal bowing (solid line) included, and with no thermal expansion.

Fig. 14 shows the beam temperature enhancements for a 9m x 6m slab with and without thermal bowing. It is interesting to note that, while the initial yield-line temperature is raised by 53°C when the beam’s thermal expansion coefficient of 1.2 x 10⁻⁵ is used rather than ignoring thermal expansion, the increase of peak beam temperature is only 11.5°C. This is reflected in other cases, and suggests that thermal deflection has little effect on peak enhancements of critical beam temperature.

5. CONCLUSIONS

On the basis of the problems identified with the derivation of the current BRE and Fracof models based on Hayes’s work in the 1960s it would clearly be unwise to use these to assess the fire resistance of composite floor panels in fire. The most significant aspect of the alternative approach, particularly in the context of the current design practice of using deformed reinforcing bars, even in the light welded meshes used as anti-cracking reinforcement, is that it includes the effect of fracture of bars across discrete, localised yield lines. In the case of more highly reinforced slabs, in which tension stiffening causes reinforcement yield to spread across a greater width, the positive anchor-points created by the weld-points to transverse bars limit the length over which reinforcement can yield, so that the fracture ductility of the reinforcement over the mesh spacing controls the fracture crack-width. Any future simplified model must obviously use the ductility of the mesh to determine the limiting deflection of the slab, but the superposition of mesh strain and thermal deflection for cases with disparate boundary conditions is meaningless. It is clearly not justifiable to use the yield-line pattern for a non-composite slab in any simplified treatment. Adding the load capacities of a concrete slab and the composite beams which form the ambient-temperature design model cannot be justified.

The approach used in the current reappraisal of tensile membrane action in composite slabs in fire is incomplete at present. The most important aspect still to be quantified is a verified calculation of fracture crack-width. The results presented in this paper are based on research [17] on reinforcement in concrete columns, but more work is needed on slabs with a range of reinforcement percentages and ductility grades, and for both plain and deformed bars.

REFERENCES


