Behaviour of Composite Floor Systems in Fire

by:

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Summary

In creating simple design tools for composite slabs in fire, a current design method has been developed with the extension of ambient-temperature tensile membrane action to the elevated-temperature phase. Experiments have since shown that the development of the mechanism at elevated temperature is aided by thermal bowing of the slab in the initial stages of the fire – a condition which is not accounted for in the ambient temperature mechanism. Numerical studies on the design method also show that increasing reinforcement ratios does not proportionally increase the capacity of composite slabs, as the method suggests. The method also assumes that vertical support, along slab panel boundaries, is maintained indefinitely throughout fire exposure. However, this support is provided by beams, which deflect under load and heat.

To help address these issues the research examines the effects of various parameters on the development of tensile membrane action at elevated temperatures. Through model-scale experiments on thin slabs, the effects of thermal gradients, acting alone, on tensile membrane action are identified. The experiments also investigate the effects of reinforcement bond strength and reinforcement ratios on the failure of concrete slabs in fire. Using a classical approach the research establishes that thermal gradients can induce a considerable amount of tensile membrane action, and quantifies the contribution this gradient makes to displacements, membrane tractions and stresses. Further studies on the effects of thermal gradients are made on axially supported slabs with different restraint stiffnesses.

An investigation into what constitutes adequate vertical slab panel support is conducted by examining various boundary support conditions, and an edge-beam failure mechanism is developed, while other possible slab panel collapse mechanisms are discussed. A comparative study on the effects of increasing reinforcement mesh on slab panel capacity is then conducted with Vulcan and the simple design method. The relationship between reinforcement area and slab spans is also established.
Acknowledgement

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The author would also like to acknowledge the support of his family, that special person who does not want to be named, all colleagues (past and present) of Room D120 and friends. A big thank you goes to Ceren, Eliot, all technicians who helped with the experimental program and all staff of the Department of Civil and Structural Engineering, at the University of Sheffield.
Declaration

Except where specific reference has been made to the work of others, this thesis is the result of my own work. No part of it has been submitted to any university for a degree, diploma or other qualification.

Anthony Kwabena Abu
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# Notation

Only general notations used in the body of the thesis are presented here. Notations that appear once, and are of a more specific nature, have been defined where they arise in the text.

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<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$A$</td>
<td>Area of reinforcement</td>
</tr>
<tr>
<td>$D$</td>
<td>Slab rigidity</td>
</tr>
<tr>
<td>$e$</td>
<td>Enhancement factor</td>
</tr>
<tr>
<td>$E$</td>
<td>Reinforcement tangent modulus</td>
</tr>
<tr>
<td>$f_y$</td>
<td>Reinforcement yield stress</td>
</tr>
<tr>
<td>$h$</td>
<td>Effective thickness of the slab</td>
</tr>
<tr>
<td>$L$</td>
<td>Longer span of the slab</td>
</tr>
<tr>
<td>$l$</td>
<td>Shorter span of the slab</td>
</tr>
<tr>
<td>$q$</td>
<td>Applied uniform floor loading</td>
</tr>
<tr>
<td>$T_0$</td>
<td>Ambient temperature</td>
</tr>
<tr>
<td>$T_1$</td>
<td>Slab top surface temperature</td>
</tr>
<tr>
<td>$T_2$</td>
<td>Slab bottom surface temperature</td>
</tr>
<tr>
<td>$T_m$</td>
<td>Slab mid-surface temperature</td>
</tr>
<tr>
<td>$w$</td>
<td>Out-of-plane deflection of the slab</td>
</tr>
<tr>
<td>$u$</td>
<td>X-direction displacement</td>
</tr>
<tr>
<td>$v$</td>
<td>Allowable vertical displacement, y-direction</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Coefficient of thermal expansion of concrete</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>Axial strain</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Shear strain</td>
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</table>
1 Introduction

The design of composite structures for fire conditions has evolved over the past 20 years. Traditional methods of prescriptively applying fire protection to all exposed steelwork after room temperature design are being replaced by performance-based approaches where the real behaviour of the three-dimensional structure, under the imposed fire conditions, is assessed in meeting the fire resistance requirement. The progress is largely due to the improved understanding of the contribution of composite slabs to overall building stability at elevated temperatures. This evolution is gradually creating an environment where higher levels of safety are maintained with massive reductions in construction costs.

1.1 Fire Resistance

1.1.1 Fire Development

Fires occur when there is a spark in the presence of oxygen and sufficient combustible material. As shown in Fig. 1.1 there are four stages in fire development. These are the incipient, growth, burning and decay stages. The incipient stage is where combustible materials heat up, marking the beginning of combustion. The fire then spreads slowly on the available combustible materials during the growth stage, and then at about 600°C, it begins to grow rapidly and enters the burning period where everything within the closed environment is aflame. The transition from the growth period to the burning period is called flashover. After all the combustible material in an environment has burned, or the fire has been starved of enough oxygen, the fire dies out in the decay period (Buchanan, 2001).

Although there are many cases where lives and property have been lost due to arson, fires normally start accidentally. The main requirement of buildings in fire situations is that they maintain a minimum level of safety for occupants to exit the building in a quick but orderly fashion. This requirement, and subsequent research that has gone into the study of fire safety, have led to the creation of the fire safety engineering discipline.
1.1.2 Fire Safety Engineering

Fire Safety Engineering is defined as:

“...the application of scientific and engineering principles to the effects of fires in order to reduce the loss of life and damage to property by quantifying the risks and hazards involved and provide optimal solutions to the application of preventive or protective measures...” (Purkiss, 1996).

This definition has characterised the design of structures for fire situations for decades. The preservation of life and property has led to the identification of certain requirements that buildings should satisfy to meet minimum fire safety. These include the control of ignition, fire detection, control of the means of escape, control of fire spread and the prevention of structural collapse (Approved Document B, Volume 2, 2007). The primary objective of fire safety engineering is to protect people; a building will ideally be designed with fire detection as well as ignition prevention systems in place, which will then be followed by a safe evacuation procedure. In some cases sprinklers may be placed in the building to prevent fires from reaching flashover. However, these active systems may, in some cases, be uneconomical, and may also be rendered ineffective or inoperative under certain circumstances. Therefore, there is the need to design passive fire systems into the fabric of buildings to compensate for any unforeseen events (Buchanan, 2001).

1.1.3 Fire resistance of building materials

Materials used in the British construction industry include concrete, steel, timber and masonry. Among these, the most preferred framing material for multi-storey...
construction is currently steel. Steel is durable, and offers quicker and lighter construction in comparison to concrete, which is renowned for its mouldable and thermal insulation properties. A major disadvantage of using steel is its low resistance to heat (Purkiss, 1996); fifteen minutes of heating a 500mm deep steel beam section to the standard temperature – time relationship (CEN, 2002) raises its temperature to about 600°C. For this reason it has been customary to protect steel elements used in construction.

In composite construction, a number of inherent advantages exist, as compared to non-composite steel-framed structures. The combination of steel and concrete materials implies the use of thinner steel sections. At elevated temperatures, the low thermal conduction of concrete helps to keep steel members below bare steel critical temperature levels, due to the non-uniform temperature distribution created. However, to satisfy building regulations, exposed steel sections in composite members are also protected so that they can meet design fire ratings, which depend on the size and use of the building (Approved Document B Volume 2, 2007). Standard fire ratings in multiples of 30 minutes have thus been developed to quantify structural performance in fire.

The fire ratings are based on testing representative isolated structural elements, loaded to levels similar to equivalent members in the structure, and then heated using the standard fire curve until failure. Failure is defined with respect to 3 main criteria:

- Insulation: temperature on the unexposed side increase by an average of 140°C or locally reaches an absolute temperature of 180°C (separating elements)
- Integrity: occurrence of cracks which enable penetration of flames from the exposed to the unexposed face (separating elements)
- Load-bearing capacity: lost if the element is no longer able to carry the applied load during the test (all elements)

Although these tests give an idea of how building elements respond to fires, they are expensive and not very representative of reality. The proper end-conditions of elements in the real structure can not be modelled accurately and large specimens can not be tested because of the limited sizes of test furnaces (Purkiss, 1996).

1.1.4 Prescriptive and performance-based methods

As described above standard thicknesses of fire protection (insulating) materials were applied to single members with exposed steelwork to give the building the required fire
resistance. This approach is intended to ensure that structural steel members never exceed temperatures of 550°C. This is because, at 600°C, steel members lose about 50% – 60% of their strength. This approach, termed the prescriptive approach, ignores specific circumstances such as the type of fire, the consequences of fire exposure, the thermal distribution through the structural member, loading conditions and the importance of different structural elements and their interactions. The introduction of codes such as BS5950-8 (BSI, 2003) and the Eurocodes (CEN, 2002; CEN, 2004; CEN, 2005a; CEN, 2005b), which allow the use of advanced calculation methods, has provided a scientific basis for the need to protect steelwork against fires. They treat fire exposure as an accidental limit state and thus provide appropriate design safety factors for loads and materials in fire conditions. They also consider material strength and stiffness reduction factors at elevated temperatures. However, they are again based on the standard fire test, and may still require the protection of all exposed steelwork in buildings with longer fire ratings.

Recent developments in research, and observations of structural behaviour under fire conditions, have shown that the design of members on the assumption of isolated behaviour in fire is over-conservative. The interactions between various parts of the three-dimensional structure often show the existence of high inherent fire resistance. Performance-based design approaches have therefore been adopted in a growing number of countries. This is a rational fire engineering approach to provide the requisite fire safety, by taking the real behaviour of the three-dimensional structure into account (Buchanan, 2001).

Determination of the fire resistance of a structure using the performance-based design approach requires consideration of fire limit state loading, thermal properties of materials at elevated temperatures, the interaction of structural elements in load-sharing mechanisms and geometrically nonlinear behaviour of structural elements at elevated temperatures.

1.2 Observed resistance of Steel-framed composite buildings
In the 1990s, a number of accidental fires and specially designed large-scale fire tests on steel-framed buildings with composite floors confirmed the over-conservativeness of the established prescriptive methods, and provided an incentive for the development of performance-based design methods.
1.2.1 Broadgate Fire

In 1990, fire broke out on the first floor of an uncompleted 14-storey steel-framed office block at the Broadgate development in London. At the time of the fire, the sprinkler system and other active measures had not been installed. Also, passive fire protection of the exposed steel beams was not complete. Fire temperatures were estimated to have reached over 1000°C, with unprotected steelwork temperatures at about 600°C. However, the integrity of the composite slab was maintained, although a separation of the steel deck from the concrete slab was observed. Deflections of composite beams were between 82mm and 270mm with a 600mm maximum deflection of the slab. There was no observed structural failure of the building, except for large distortions in the form of local buckling of the bottom flanges of beams near their supports and the shortening of smaller columns; all were primarily due to the restraint to thermal expansion of these parts by other parts of the structure, which were at considerably lower temperatures (Newman et al., 2000; 2006).

1.2.2 Churchill Plaza building, Basingstoke

Churchill Plaza, built in 1988, was a 12-storey building which housed an insurance company in Basingstoke, UK. In 1991, a fire started on the 8th floor and spread rapidly to the 10th floor through glazing failure. Incidentally there was on-going maintenance work on the 8th and 9th floors, so the sprinklers were inoperative. However they were fully functional on the 10th floor. The building had been designed for a 90-minute exposure to the standard fire, so no permanent structural frame deformations were observed, although separation of the steel deck and the concrete slab occurred. After the fire the most badly affected area of the concrete slab was tested with 1.5 times its original design load. It was observed that the slab had adequate residual load-carrying capacity and could be used without repair (Newman et al., 2000; 2006).

Tests on composite floors in Australia have also shown that the use of unprotected intermediate beams with sprinkler systems that have sufficient reliability provide a higher level of safety than similar constructions with traditional prescriptive protection, if a non-fire-rated false ceiling is used to help reduce the thermal exposure of the floor beams (Newman et al., 2000; 2006).

1.2.3 Cardington Tests

The Broadgate fire had shown that, although some structural elements lost their load-bearing capacity in fire, the composite slab, with its supporting steelwork and other cooler parts of the building, positively influenced the stability of the structure by acting
as a membrane to distribute loads away from the weakened members. Churchill Plaza had provided confirmation of the over-conservative nature of the traditional design method of protecting all exposed steelwork. The load test performed on the most degraded part of the slab had shown that an excess capacity existed, but this capacity could not be quantified. The Australian tests had shown what actually happens in real buildings. However, any reliance on suspended ceilings which are not fire-rated does not allow for easy quantification of the advantages of applying this procedure in practice.

The inadequate representation of structural elements by the standard fire test and the observations of real building fires led to the development of several fire tests at the Building Research Establishment (BRE)'s Large Building Test Facility at Cardington between 1995 and 2003. In all, seven tests were performed on a specially-designed 8-storey steel-framed building; six tests were performed between January 1995 and July 1996, while the seventh was performed in January 2003. The tests demonstrated the behaviour of real structures under fire conditions, and provided test data for the development and calibration of computer programs. The building had a floor footprint of 21m x 45m and an overall height of 33m, with a 9m x 2.5m central lift core and two stair-wells, braced to resist lateral loads. It was designed to BS5950 (BSI, 1990a; 1990b), and checked against Eurocodes 3 and 4 (CEN, 1992; CEN,1994) for compliance, using S275 and S355 hot rolled steel sections and a 130mm deep composite slab on a 0.9mm thick PMF CF70 steel deck. The slab was made of lightweight Grade 35 concrete, reinforced with a standard A142 mesh. Two views of the structure are presented in Fig. 1.2. Live loading was simulated with sandbags, each weighing 11kN, giving an overall floor loading of 5.48kN/m².

Fig. 1.2: Cardington test building (Foster et al., 2007)
The first six tests were on: a restrained beam; a plane frame; two corner compartments; a large compartment and an office fire demonstration. The seventh test was conducted to collect more data on the behaviour of beam-to-beam and beam-to-column connections, especially in the wake of the collapse of the World Trade Centre towers. The test also provided the opportunity to check the suitability of specialised numerical modelling software (Foster et al., 2007). The locations of the tests are shown in Figs. 1.3 and 1.4.

Fig. 1.3: Locations of the first 6 Cardington tests (Foster et al., 2007)

Fig. 1.4: Location of the 7th Cardington test (Foster et al., 2007)
The tests established that, for stability of a structure in fire, structural damage should be limited to fire compartments of origin, and therefore recommended the protection of the entire lengths of columns in any fire compartment although steel beams were left unprotected. Beam temperatures reached between 800°C and 1150°C (well above their limiting design temperature of 680°C), and with maximum slab displacements between 232mm and 641mm. It was concluded that, although most floor beams lost strength and stiffness, flexural bridging at relatively small deflections and membrane action of the composite beams and slabs at large deflections introduced structural stability and alternative load paths. Catenary action of beams and slabs bending in single curvature contributed to their enhanced capacity at large deflections. The ability of the composite slabs to bear considerably higher loads at large deflections and in biaxial bending was attributed to tensile membrane action (Martin and Moore, 1999; Huang et al., 2002b; Foster et al., 2007).

1.3 Tensile membrane action
The results from Cardington indicated that the existing design methods for composite slabs in fire were too conservative, and therefore design methodologies which incorporated the observed higher load-bearing mechanisms (tensile membrane action and catenary action) were needed to take advantage of the inherent fire resistance within the fabric of steel-framed composite structures. Tensile membrane action is a load-bearing mechanism of thin slabs undergoing large vertical displacement, where the induced radial tension in the centre of the slab is sustained by a peripheral ring of compression. A diagrammatic representation of tensile membrane action is shown in Fig. 1.5.
A vertical deflection of the order of the thickness of the slab marks the incidence of this mechanism. The self-sustaining nature of tensile membrane action implies that the process occurs with or without horizontal restraint once the basic requirement of biaxial bending and vertical edge supports are satisfied.

The observations of steel-framed building behaviour at elevated temperatures, from accidental fires and especially from the Cardington tests, have led to the development of several simplified design methods for the determination of composite slab capacities in fire, incorporating tensile membrane action (Bailey, 2000; Clifton, 2001; Cameron and Usmani, 2005a; Omer et al., 2006; Li et al., 2007). Of these, the Bailey-BRE method has been adopted by the Steel Construction Institute (SCI), and design guidance on the use of this method in generating higher load-bearing capacity has been developed (Newman et al., 2000; 2006).

1.4 Bailey-BRE Method
To observe the actual behaviour of composite floor systems in fire, the bulk of the Cardington fire tests were conducted with applied fire protection to columns - leaving all beams unprotected; the only exception to this protection scheme was the British Steel Corner Test, which had protection applied to its edge beams (Martin and Moore, 1999). In certain cases vertical support, at the tested compartment edges connecting with the building perimeter was provided by vertical ties, known as wind-posts. In other areas compartment walls were built beneath the edge beams. The observed increase in slab capacity was attributed to tensile membrane action due to biaxial bending, and the compartment perimeter vertical support, which was provided by the vertical ties or the use of protected beams. This observation was similar to those made by early researchers on tensile membrane action at ambient temperature (Wood, 1961; Park, 1964; Taylor, 1965; Sawczuk and Winnicki, 1965; Kemp, 1967; Hayes, 1968). The Bailey-BRE design method was therefore pioneered by Bailey and Moore (2000a; 2000b) as a use of this mechanism in the design of composite slabs in fire conditions.

A 9.5m x 6.5m slab test (Bailey et al., 2000) was performed at ambient temperature to simulate elevated temperature conditions, due to the complications of performing full-scale fire tests. A transverse full-depth tension crack was observed in the middle of the slab at failure after a load of 4.81kN/m² had generated vertical deflections over 650mm. Issues that were identified from this ambient-temperature evaluation of the elevated-temperature mechanism included:
• the effect of the reinforcement on the fire resistance of the slab when its temperatures exceed 400°C
• the effect of thermal curvature and non-linear thermal gradients through the slab depth
• the effect of the surrounding cold structure on slab behaviour at elevated temperatures
• the relevance of the failure mode in actual buildings, including the effects of the steel deck and the secondary beams.

These concerns were addressed in the development of the design method.

1.4.1 Development of the Design Method
Previous investigations of membrane action at ambient temperature had shown that patterns of yield-lines remained unchanged as the vertical displacement increased (Wood, 1961; Hayes, 1968). BRE’s ambient temperature test also showed good correlation with previous research (Bailey, 2000), with a large tensile crack eventually forming across the shorter span. The earlier research had shown that there are two possible modes of failure; the more critical being the formation of transverse cracks across the short span at the intersections of the yield lines (failure mode \(i\) in Fig. 1.6). Following the observations of the BRE Garston test and other slab tests on tensile membrane action, Bailey adopted the second mode of failure for the design method.

![Mode of failure (i)](image1)

![Mode of failure (ii)](image2)

**Fig. 1.6: Failure modes identified by Sawczuk and Winnicki (Bailey, 2001)**

The Bailey-BRE method proceeds by dividing a composite floor into several horizontally-unrestrained rectangular fire-resisting zones of low aspect ratio, called slab panels (Fig. 1.7). These are composed internally of simply-supported unprotected beams (Bailey and Moore, 2000a). With increasing exposure to elevated temperatures, the formation of plastic hinges in the unprotected beams re-distributes the loads to the two-way bending slab, undergoing large vertical deflections.
Based on rigid-plastic theory with large change of geometry, and following a similar procedure to that derived by Hayes (1968), the additional slab capacity provided by the in-plane stresses (shown in Fig. 1.8) is calculated as an enhancement to the traditional small-deflection yield-line capacity.

The original Bailey-BRE method defined failure as the formation of a full-depth central tension crack across the shorter span of the slab (Bailey, 2000; 2001; 2003; 2004). In a recent revision of the method, failure has been re-defined as either the tensile fracture of reinforcement across the shorter span of the slab or the compressive crushing at its corners (Bailey and Toh, 2007a). The amendments to the method include a trapezoidal stress distribution along the diagonal yield-lines (as shown in Fig. 1.8) and the recognition of an area of compression at the edge of the slab, instead of the single point in the original method.

The method conservatively ignores any contribution of the tensile strength of concrete to the capacity of the slab. It is assumed that in practice applied fire protection will provide the necessary perimeter vertical support along the slab panel boundaries. The capacity of protected secondary beams is checked for increased load at elevated temperatures.
Chapter 1: Introduction

To predict the ultimate failure at the fire limit state a vertical displacement limit, derived from a combination of thermal bowing of the slab and the mechanical strain in the reinforcement, is defined as shown in Equation 1.1. The equation was calibrated against the Cardington fire tests. The deflection due to mechanical strain of the reinforcement is limited to $\frac{l}{30}$, where $l$ is the length of the shorter span of the slab panel. A full derivation of the method, and its recent modifications, for both isotropic and orthotropic reinforcements can be found in the references (Bailey, 2000; 2001; 2003; 2004; Bailey and Toh, 2007a).

\[
\begin{align*}
\phi &= \frac{k(1-\nu)l_d}{1+k} \\
T_1 &= bKT_o \left(1 - 2n\right) L \\
T_2 &= bKT_o \left(vL_d\right) \\
T_2' &= \frac{bKT_o}{2(1+k)}(1-\nu)L_d \\
C &= \frac{k^2bKT_o}{2(1+k)}(1-\nu)L_d \\
\end{align*}
\]

where $L_d = \sqrt{(nL)^2 + l^2/4}$

**In-plane forces:**

- Yield force $= T_o$
- Ultimate force $= f_t T_o$
- Yield moment $= M_o$

**Reinforcement:**

- Yield force $= KT_o$
- Ultimate force $= f_t KT_o$
- Yield moment $= \mu M_o$

**Fig. 1.8: In-plane stress distribution for the Bailey-BRE Method (Bailey and Toh, 2007a)**

To predict the ultimate failure at the fire limit state a vertical displacement limit, derived from a combination of thermal bowing of the slab and the mechanical strain in the reinforcement, is defined as shown in Equation 1.1. The equation was calibrated against the Cardington fire tests. The deflection due to mechanical strain of the reinforcement is limited to $l/30$, where $l$ is the length of the shorter span of the slab panel. A full derivation of the method, and its recent modifications, for both isotropic and orthotropic reinforcements can be found in the references (Bailey, 2000; 2001; 2003; 2004; Bailey and Toh, 2007a).

\[
v = \frac{\alpha(T_2 - T_1)l^2}{19.2h} + \sqrt{\frac{0.5f_{y,\theta}(Y)}{E_{y,\theta}}} \times \frac{3L^2}{8}
\]  

(1.1)

where: $\alpha$ is the coefficient of thermal expansion of the concrete slab
- $T_2$ and $T_1$ are the bottom and top surface temperatures of the slab;
- $L$ and $l$ are the longer and shorter spans of the slab panel;
- $h$ is the effective thickness of the slab
- $f_{y,\theta}$ and $E_{y,\theta}$ are reinforcement Strength and Young’s modulus at a given time
The composite slab capacity at any given time in fire is calculated as:

$$w_{\theta} = e \left( \frac{\text{Internal work done by the composite slab in bending}}{\text{External work done by the applied load per unit load}} \right) + \frac{\text{Internal work done by the beams in bending}}{\text{External work done by the applied load per unit load}}$$

(1.2)

where: $w_{\theta}$ is the slab panel capacity at a given time

$e$ is the enhancement of the slab capacity, calculated as in the reference (Bailey and Toh, 2007a)

### 1.4.2 SCI P288 and TSLAB

To facilitate the use of the Bailey-BRE method in the United Kingdom, the Steel Construction Institute (SCI) prepared a design guide (P-288), which provides guidance on the use of the method and lists tables of minimum reinforcement mesh sizes required to satisfy the allowable Bailey-BRE deflection limit criterion (shown in Equation 2.12) at a defined fire resistance time. The original document by Newman et al. (2000) has been revised with tables for a fire resistance up to 120 minutes, using the standard temperature-time relationship (Newman et al., 2006).

The required reinforcement mesh sizes are based on the type of concrete, the slab panel geometry and the type of steel decking used. In addition to the design tables, the SCI has developed a Microsoft Excel-based spreadsheet called TSLAB (Newman et al., 2006). This tool determines whether the reinforcement selected for particular slab panel geometries will be satisfactory, and includes all the advances which have been incorporated into the method recently, whilst the tables in SCI document P-288 serve as a basic guide in choosing the minimum reinforcement for any given geometry.

TSLAB has been developed as an extension to the basic Bailey-BRE membrane action method. It begins by performing thermal analyses on the unprotected intermediate beam and the composite slab. Then, using the temperatures of the individual components and its allowable vertical deflection criterion (Equation 1.3), it calculates the total capacity of the simply-supported slab panel model (by summation of the residual unprotected beam capacity and the enhanced slab capacity). The difference between Equation 1.3 and Equation 1.1 is that the cold reinforcement properties are maintained at elevated temperature.
\[ v = \frac{\alpha(T_2 - T_1)^2}{19.2h} + \left( \frac{0.5 f_y}{E} \right)_{\text{Re inf t=20°C}} \times \frac{3L^2}{8} \]  

(1.3)

This capacity is then checked against the applied load in the Fire Limit State. If the capacity of the panel is found to be below the applied load at the fire limit state, then either the resistance of the internal beams or the reinforcement mesh size must be increased (Newman et al., 2006).

Since the initial development of the Bailey-BRE method, attempts have been made by various researchers to enhance the design approach through experimental, analytical and numerical approaches.

1.5 Research on the Bailey-BRE method and tensile membrane action at elevated temperatures

Clifton (2001) expanded the initial Bailey-BRE method to include the effects of continuity and additional reinforcement that may be present in the ribs of slabs. His method also includes the capacity of the heated unprotected beams in its yield-line calculation. It imposes no limitation on slab size, and checks individual components of a slab panel, such as protected beams and columns. Subsequent to Clifton’s method, Bailey investigated the contribution of catenary action of the unprotected beams to the load capacity of the slab panel, and found that this was negligible (Bailey, 2004).

Usmani and Cameron (2004) developed a 3-step procedure for the calculation of the capacity of laterally restrained composite slabs at elevated temperatures. The motivation came from results of numerical and analytical studies after the Cardington tests, which suggested that the behaviour of composite slabs at elevated temperatures was dominated by thermal strains rather than gravity loading. It was observed that thermal bowing of the slab produced a deflected shape that was conducive to tensile membrane action. Therefore, on the assumption that at large displacements (and in fire) the bending resistance of a composite slab becomes negligible, the method assumes that failure occurs by fracture of reinforcement along axially restrained and vertically supported slab boundaries. The temperature distribution over the depth of the slab is estimated for a given fire scenario, the deflected shape of the slab and the corresponding thermally-induced membrane stresses and strains are determined, and then an energy method is used to determine the maximum load that the slab can bear.
Due to the expense of performing full-scale composite slab tests in fire, a number of finite element programs have been developed to simulate slab behaviour at elevated temperatures. These include *Vulcan* (Bailey, 1995; Huang et al., 2003a; 2003b; 2004a), a finite element program developed by the University of Sheffield for the analysis of steel and concrete structures; ADAPTIC (Izzuddin and Elghazouli, 2004a; 2004b; 2004c; 2004d), developed by Imperial College (London); SAFIR, developed at the University of Liege (Belgium) and ABAQUS 6.5, a general-purpose finite element code (Wang, 2002). These finite element codes have been used by various researchers in the verification of experiments, and in analytical studies of tensile membrane action at elevated temperatures.

SAFIR was used by Lim (2003) to investigate compressive and tensile membrane action behaviour of one- and two-way supported slabs in fire. The study, conducted in conjunction with experiments (Lim and Wade, 2002) showed that axial restraints affect the development of compressive membrane action in one-way slabs, while the tensile capacity of concrete is significant in a proper evaluation of tensile membrane action in concrete slabs at elevated temperatures (Lim et al., 2004a; 2004b).

Foster (2006) examined the behaviour of concrete slabs at large deflections, at room temperature and in fire, by performing tests on small-scale slabs. The study showed that there was generally a good correlation at ambient temperature with the Bailey-BRE predictions (Foster et al., 2004). At elevated temperatures, it was observed that, contrary to the assumption of the Bailey-BRE method, the transverse crack across the shorter span formed first without any clear indication of failure, before yield-lines appeared at failure when material strengths had reduced considerably. It was also observed that using reinforcement with low bonding to the concrete generated higher slab capacities. The formation of large deflections before clear failure of the slabs suggested a high influence of thermal gradients in the initial stages of fire exposure (Foster, 2006).

Along with Foster’s experiments, other studies were conducted with *Vulcan* to examine the influence of reinforcement ratios and various protection regimes on slab capacities in fire. The investigations showed that the Bailey-BRE method predicts a proportionally higher capacity with increasing reinforcement size, while numerical simulations and experiments suggest otherwise (Foster, 2006, Huang et al., 2000; 2001a; 2001b; 2002a).
The extensive work undertaken by various researchers prompted Bailey and Toh (2007a; 2007b) to perform a series of experimental analyses on small-scale slabs at ambient and elevated temperatures. In general, the tests show that ductile reinforcement was suitable for tensile membrane action, as very large deflections could be sustained without failure. The ambient-temperature tests also showed two failure modes; tensile fracture of reinforcement in the middle of the slab when lower reinforcement ratios were used, and compressive crushing at the slab corners when reinforcement ratios were high. Although compressive failure was not observed during the elevated-temperature tests, the test results have helped in the re-development of the Bailey-BRE method (Bailey and Toh, 2007a; 2007b).

The reliance of the Bailey-BRE method on the calculation of composite slab capacities as enhancements to the traditional yield-line load, and the extension of the mechanism to elevated temperatures introduce a number of serious issues. These include the effects of reinforcement ratios, thermal gradients and the assumption of vertical support along the slab panel boundary. The research outlined in this thesis therefore seeks to address some of these issues to help quantify composite slab capacities in fire.

1.6 Objectives and Thesis Outline
The present study examines the true development of tensile membrane action from a purely thermal perspective. The thesis investigates the behaviour and failure of concrete slabs through experiments in Chapter 2. Chapter 3 examines the effects of thermal gradients on the development of the tensile membrane mechanism through a classical approach, while Chapter 4 extends this study on slab behaviour at elevated temperatures by examining the effects of different boundary conditions and thermal stresses through the depth of the slab. Chapters 5 and 6 investigate the effects of slab panel edge support and reinforcement ratios on panel capacity, while making the necessary comparisons with the Bailey-BRE method. Conclusions and recommendations for future work are then made in Chapter 7.
Chapter 2: Experimental studies on failure of thin slabs in fire

2 Experimental studies on failure of thin slabs in fire

The elevated temperature tests performed by Foster (2006) examined two slab aspect ratios (1.55 and 1.00) with smooth and deformed reinforcement at either the mid-plane level or 7.5mm from the bottom of the slab. In all the tests, the load ratios were above 1.0. Although these higher load ratios allowed the visualisation of yield-lines at failure, the test setup made it difficult to rationalise the results. In practice, slabs will normally not be loaded to their limit. Consequently, in fire, the loss of some imposed loading will result in reduced load ratios.

The experiments in this chapter were therefore designed to follow Foster’s tests, but to evaluate the failure of concrete slabs with varying reinforcement ratios and bond characteristics while maintaining the load ratio at 1.0. Other tests were conducted with a load ratio of 2.0, to observe the likely failure modes as reinforcement ratio increases. The tests also examined the influence of thermal gradients, acting alone, on the mobilisation of tensile membrane action. For adequate interpretation, the tests have been compared with the revised Bailey-BRE method and *Vulcan*.

2.1 Elevated temperature tests

Thirteen elevated-temperature tests were performed on concrete slabs of nominal dimensions 900mm x 600mm x 15mm. The slabs were made from gritty sand with fine aggregate between 1mm - 4mm in size, a cement-sand ratio of 1:3 and a water-cement ratio of 0.47. Each concrete slab was reinforced with either smooth or deformed mild steel tying wire of either 0.71mm or 0.9mm diameter. The deformed wires were made by passing the smooth wire though a purpose-built machine (see Fig. 2.1).

The strengths and ductility characteristics of the smooth and deformed wires were determined by performing tensile tests with a tensometer. As the characteristics of the 0.71mm diameter wires had been assessed already (Foster, 2006), the results shown in Fig. 2.2 are for the 0.9mm diameter wires. From Fig. 2.2, it can be observed that the smooth reinforcing wire had yield strengths between 231 - 314N/mm² with the yield strength of the deformed wire varying between 226 - 304N/mm². The ultimate strengths were between 305 - 395N/mm² for deformed wires and 290 - 375N/mm² for smooth wires.
Fig. 2.1: Deforming machine with a deformed wire

Fig. 2.2 shows that lower strengths were recorded in the first series of tensile tests. This was because there was considerable slippage, as the wires were not properly secured in the grips. More consistent results were obtained after the wires had been wound round the grips several times. The adopted average ultimate reinforcing wire strengths were therefore 380N/mm$^2$ for the smooth wires and 360N/mm$^2$ for the deformed wires (by considering results from the 6th test onwards).

![Graph showing tested strengths of smooth and deformed wires](image)

**Fig. 2.2: Tested strengths of the smooth and deformed wires**

Tested ductility values for the smooth and deformed wires are shown in Figs. 2.3 and 2.4 respectively. The tests show that the smooth reinforcement had a ductility of 28.0% while the ductility of the deformed wire was found to be 10.6%.
Before fresh concrete was cast, the reinforcing mesh was prepared by tying either smooth or deformed wires (in two orthogonal directions) to sets of nails on a wooden frame, after the wires had been cleaned with acetone to remove grease, rust and dirt. The nails were set at 5mm spacing; this allowed the reinforcing mesh to be spaced at multiples of 5mm. The mesh was set such that the bottom reinforcement spanned in
the shorter direction of the slab (see Fig. 2.5). The reinforcement ratios used in the slabs ranged from 0.1% to 0.5% by cross-sectional area; only isotropic reinforcement was used.

![Fig. 2.5: Wooden frame with reinforcing mesh](image)

The prepared reinforcing mesh was placed and fastened onto a prepared slab mould so that the reinforcement was located at the mid-depth of the 15mm thick slab. The mould was prepared for each cast by cleaning and then lubricating with commercial wax.

Casting was performed in conjunction with vibration on a vibrating table to ensure that the 15mm nominal thickness was achieved throughout the slab. Concrete cubes of dimensions 100mm x 100mm x 100mm were cast in addition to the slab, so that concrete strengths at 28 days, and on the test date, could be evaluated. On each casting day, two slabs could be prepared; therefore, nine concrete cubes were cast with these slabs so that three concrete cubes were used for each to assess the strengths on the 28th day and the two test dates. The cubes and the fresh concrete slab were cured by covering with damp hessian for 24 hours before they were taken out of the mould for air-curing until the days of the tests.

Before each test, the average slab thickness and the effective depth of reinforcement were determined by taking the averages at several locations. This process was necessary to calculate its capacity, so that an accurate load could be placed on it. Thermocouple and transducer positions were marked on the slab. The thermocouples measured temperatures of the top surface, the bottom surface and the reinforcement, and these were continuously monitored throughout the test. Four thermocouples were therefore placed at each level, after drilling to the appropriate depth. Seven
transducers were used; four measuring in-plane displacement and three measuring vertical displacement. Twelve loading positions were also marked. These loading positions allowed the applied load to simulate a uniformly distributed load. The loading positions, transducer locations and the levels at which temperatures were monitored are shown in Fig. 2.6.

The concrete slab was mounted on a support frame and lightly clamped at its corners to resist curling-up. The rectangular support frame (see Fig. 2.7(a)) had smooth circular rods welded onto it to provide a horizontally-unrestrained but vertically-supported slab with a loading area of 850mm x 550mm. The simulation of uniformly distributed load by the 12 load points, shown in Fig. 2.6, is therefore based on this loading area. The support frame was placed over a rectangular furnace, which had four electrical heating elements, capable of generating temperatures up to 700°C (see Fig. 2.7(b)). Loads were applied through a hydraulic pump. The load was then distributed to the 12 loading points though the use of v-grooved bars and steel balls to triangular loading plates, such that the vertical loading was maintained throughout the test by allowing the alteration of the relative positions of the balls in the v-groove.

The use of the hydraulic pump required that the applied load was kept at a constant value throughout the test. This was necessary because the increased deflections of the slab would effectively reduce the value of a rigidly applied load, by reducing the pressure in the pump.
The interior of the furnace was lined with 50mm of Kaowool insulation to prevent heat losses, and to concentrate the heat on the underside of the slab. Fire blankets were also used to prevent heat losses through openings along the vertical support frame as the slab deflected in a test.

The various tests conducted are shown in Tables 2.1 and 2.2. In the tables, slabs reinforced with either smooth or deformed reinforcing wires are denoted by SM and DF, respectively.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Wire Diameter (mm)</th>
<th>Reinforcement ratio</th>
<th>Wire Type</th>
<th>Cube Strength (N/mm²)</th>
<th>Slab Thickness (mm)</th>
<th>Reinforcement depth (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.71</td>
<td>0.1%</td>
<td>SM</td>
<td>44.0</td>
<td>15.0</td>
<td>7.50</td>
</tr>
<tr>
<td>2</td>
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<td>0.1%</td>
<td>DF</td>
<td>46.0</td>
<td>15.0</td>
<td>7.50</td>
</tr>
<tr>
<td>3</td>
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<td>0.5%</td>
<td>DF</td>
<td>41.0</td>
<td>15.0</td>
<td>7.50</td>
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<tr>
<td>4</td>
<td>0.90</td>
<td>0.2%</td>
<td>SM</td>
<td>53.4</td>
<td>14.2</td>
<td>8.05</td>
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<tr>
<td>5</td>
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<td>0.2%</td>
<td>DF</td>
<td>53.5</td>
<td>14.2</td>
<td>7.85</td>
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<tr>
<td>6</td>
<td>0.90</td>
<td>0.1%</td>
<td>DF</td>
<td>42.3</td>
<td>13.6</td>
<td>7.85</td>
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<tr>
<td>7</td>
<td>0.90</td>
<td>0.1%</td>
<td>DF</td>
<td>39.0</td>
<td>13.6</td>
<td>7.15</td>
</tr>
<tr>
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<td>0.90</td>
<td>0.4%</td>
<td>DF</td>
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<td>14.2</td>
<td>7.65</td>
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<td>0.4%</td>
<td>SM</td>
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<td>SM</td>
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<td>13.5</td>
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<td>DF</td>
<td>55.4</td>
<td>13.6</td>
<td>7.35</td>
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<td>12</td>
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<td>0.4%</td>
<td>SM</td>
<td>47.1</td>
<td>14.2</td>
<td>7.35</td>
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<tr>
<td>13</td>
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<td>0.4%</td>
<td>DF</td>
<td>50.9</td>
<td>15.6</td>
<td>8.55</td>
</tr>
</tbody>
</table>

Table 2.1: Geometric and material properties of the various tests
Chapter 2: Experimental studies on failure of thin slabs in fire

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Test Date</th>
<th>Wire Diameter (mm)</th>
<th>Reinforcement ratio</th>
<th>Wire Type</th>
<th>Yield Capacity (kN/m²)</th>
<th>Ultimate Capacity (kN/m²)</th>
<th>Applied Loading (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>05-08-2005</td>
<td>0.71</td>
<td>0.1%</td>
<td>SM</td>
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<td>1.82</td>
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<td>0.1%</td>
<td>DF</td>
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<td>0.71</td>
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<td>DF</td>
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<td>8.22</td>
<td>16.12</td>
</tr>
<tr>
<td>4</td>
<td>11-04-2006</td>
<td>0.90</td>
<td>0.2%</td>
<td>SM</td>
<td>4.02</td>
<td>5.02</td>
<td>5.02</td>
</tr>
<tr>
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<td>0.5%</td>
<td>DF</td>
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<tr>
<td>6</td>
<td>13-04-2006</td>
<td>0.90</td>
<td>0.1%</td>
<td>DF</td>
<td>2.13</td>
<td>2.72</td>
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<tr>
<td>7</td>
<td>19-04-2006</td>
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<td>0.1%</td>
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<td>2.49</td>
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</tr>
<tr>
<td>8</td>
<td>24-04-2006</td>
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<td>0.2%</td>
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<tr>
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<td>0.4%</td>
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<td>13</td>
<td>03-05-2006</td>
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<td>0.4%</td>
<td>DF</td>
<td>7.98</td>
<td>9.94</td>
<td>19.88</td>
</tr>
</tbody>
</table>

Table 2.2: Test properties of the various tests

2.2 Results

As the lists of tests in Tables 2.1 and 2.2 show, the tests were carried out in two phases; the first was performed with 0.71mm reinforcing wire while the second was with the 0.9mm reinforcing wire. Before the commencement of each phase, a practice test was performed to observe temperature distributions in the furnace, and to test and calibrate the instrumentation used (i.e. the thermocouples, the transducers and the hydraulic jack). Typical temperature distributions at the bottom surface, the top surface and the reinforcement levels are shown in Fig. 2.8. Temperatures at the bottom surface, the reinforcement level and the top surface are represented by B, R and T respectively.

![Fig. 2.8: Typical temperature distribution through the slab cross-section](image)
2.2.1 Test 1

The first test was performed on 5\textsuperscript{th} August 2005. The 900mm x 600mm slab was reinforced with 0.1\% smooth wire of diameter 0.71mm. As the effects of thermal gradients were being investigated, the slab was unloaded. At a relatively low deflection of 2mm, cracks resembling corner cracks were seen on top of the slab, away from the corners (see Fig. 2.9). This was not typical of the experiments conducted by Foster (2006); in which the corner cracks were very close to the corners. This difference in behaviour was therefore attributed to thermal effects only. Transverse cracks started appearing when the central deflection reached 5.21mm, with bottom surface temperatures at 275\degree C. These cracks became more visible at a vertical deflection of 8.24mm with a corresponding bottom surface temperature of 464\degree C.

From Fig. 2.9 it is seen that the transverse tension cracks were only visible on the top surface of the slab, and not on the bottom face. No yield-lines were seen either during or after the test. Plots of the various vertical and horizontal displacements of the slab with time are shown in Fig. 2.10 and Fig. 2.11, respectively.

The graphs confirm the occurrence of tensile membrane action. Fig. 2.10 shows that a central vertical displacement of 14.6mm was recorded at about 117 minutes into the test. The horizontal displacements in Fig. 2.11 show that the longer edges of the slab were pulled inwards while those on the shorter edge were pulled outwards. The displacements are all shown as positive (outwards) because of the thermal expansion of the slab.
2.2.2 Test 2

The second test was also unloaded. In this test, however, the slab was reinforced with 0.1% of deformed reinforcement of 0.71mm diameter. Similar to Test 1, corner cracks appeared at a displacement of 2.5mm. Again, these cracks were away from the corners. A transverse crack, which became visible at a displacement of 5.2mm during the test, was observed to have penetrated the full depth of the slab, after the specimen had been removed from the test furnace (see Fig. 2.12).
The test was terminated early, when temperatures became virtually static for long periods of time, probably due to a lack of adequate insulation along the top edges of the furnace, allowing some of the heat to escape. There were no indications of yield lines in this test. The through-depth tension crack can be attributed to the tensile stresses induced in the concrete by the differential thermal expansion through the depth of the slab. The maximum vertical deflection recorded was 6.07mm (Fig. 2.13). Although this does not clearly indicate the occurrence of tensile membrane action, subsequent modelling with *Vulcan* showed that the mechanism occurred (Section 2.3.1, Fig. 2.39). A plot of the horizontal displacements of the slab edges shows the trend observed in Test 1, with smaller outward movement of the longer edges compared with the larger movement of the shorter edges (Fig. 2.14).
2.2.3 Test 3

To test the effects of higher reinforcement ratios, Test 3 examined a 0.5% reinforcement ratio using 0.71mm diameter deformed wires. An attempt was made at loading the slab to twice its yield-line load. As the applied load was equally distributed to the 12 loading points, the load required in addition to the slab self-weight (0.156kN) and the loading equipment (0.179kN) was 7.2kN. However, this was reduced to 4.11kN after crushing of concrete, at the slab corners, was observed at about 46 minutes into the test.

The load was further reduced to 3.46kN before the early termination of the test at 70 minutes, at which point the central vertical displacement had reached 74.5mm. In addition to corner crushing, large deflections and cracks occurred beneath the loading points - parallel to the length of the slab (Fig. 2.15). There were no transverse cracks. Although yield lines could not be seen during the test, distributed crack patterns following the trend of yield lines were seen on the bottom surface, after the slab was removed from the furnace (Fig. 2.15). The reinforcement in these corners of crushed concrete did not fracture.

The slab behaviour was therefore attributed to compressive failure, as the high amount of reinforcement provided little room for the development of a larger compressive ring at larger deflections, thereby causing failure of the slab. Compressive failure was
confirmed by further analyses with *Vulcan* (Section 2.3.4). Slab vertical displacements, recorded during the test, are shown in Fig. 2.16.

**Fig. 2.15: Top and bottom slab faces in Test 3**

*Fig. 2.16: Vertical displacements in Test 3*

### 2.2.4 Test 4

This was the first test in which 0.9mm diameter reinforcing wires were used. A load of 5.02kN/m², equivalent to the ultimate capacity of the slab at small deflections, was imposed on the slab, reinforced with 0.2% smooth wire. Corner cracks appeared during ambient temperature loading at a vertical displacement of 2.15mm. As with all tests in this series, the applied loading was maintained for a minimum of 60 minutes.
before the heating elements were turned on. A single transverse crack was observed at about 10-15 minutes into the heating phase before the formation of yield lines towards the end of the 2-hour fire test. The top and bottom slab surfaces are shown in Fig. 2.17.

![Top and bottom slab faces in Test 4](image)

The transverse crack and the yield lines penetrated the full depth of the slab, as shown in Fig. 2.17. It was observed that the transverse crack was aligned towards the intersection of the diagonal yield lines, as suggested by Sawczuk and Winnicki (1965). Bottom surface temperatures reached 638°C, with a corresponding maximum vertical displacement of 51.9mm. A plot of the vertical displacements with time is shown in Fig. 2.18.

![Vertical displacements in Test 4](image)
2.2.5 Test 5

Test 5 was modelled on Test 4. The difference between the two tests was the type of reinforcing wire used. A reinforcement ratio of 0.2% of deformed wire was used in this test. The applied load was 4.64kN/m², equivalent to the theoretical small-deflection slab capacity. The deformed wires used in this test allowed the formation of more visible cracks. Again, corner cracks appeared first, followed by three transverse cracks. One was considerably larger than the other two and was oriented towards the centre of the slab. There were no distinct yield lines, although there were small indications of their formation. Only the large transverse crack was visible from the slab bottom surface, after the test (see Fig. 2.19). There were no visible signs of failure, but the slab reached a central vertical displacement of 40mm (Fig. 2.20) at a bottom surface temperature of 593°C.

![Fig. 2.19: Top and bottom slab faces in Test 5](image)

![Fig. 2.20: Vertical displacements in Test 5](image)
2.2.6 Test 6
This test used 0.1% of deformed reinforcement of 0.9mm diameter. It was run for four and half hours. It ended when a steady deflection was reached without considerable increase in temperature. As with the previous loaded tests, the test load was at a ratio of 1.0 of the theoretical small-deflection capacity of the slab. Corner cracks were recorded at ambient temperature. A small transverse crack was observed developing in the centre of the slab shortly after the heaters were switched on. The top and bottom surfaces of the slab are shown in Fig. 2.21.

![Top and bottom slab faces in Test 6](image)

There were no clear signs of failure. Yield lines were confirmed by examining the bottom surface of the slab, after the test specimen had been removed from the furnace. A maximum central deflection of 24.2mm was attained at 270 minutes. Vertical displacements recorded during the test are shown in Fig. 2.22.

![Vertical displacements in Test 6](image)
2.2.7 Test 7

Test 7 was the first test conducted with a load twice the theoretical limit capacity of the slab, in the second series of experiments. The use of such high loading was to assess the influence of reinforcement ratios on failure modes of concrete slabs at elevated temperature. This investigation was deemed necessary to understand the failure pattern that was observed in Test 3 with the 0.5% deformed wire reinforced slab. The top view of the slab, after the test, is shown in Fig. 2.23.

![Top face of slab in Test 7 and the transverse crack](image)

During ambient-temperature loading, yield lines started appearing shortly after corner cracking. The formation of the transverse crack was then detected at a deflection of about 10mm. This crack expanded with increasing vertical displacements. The ambient-temperature load was maintained for 60 minutes before the elevated temperature phase began. The deflection recorded before the start of the heating phase was 18.4mm. Due to an inadequate setup of the central displacement transducer, the maximum vertical displacement that was recorded was 70mm (see Fig. 2.24). The test was terminated after this occurred. The width of the transverse crack was about 15mm. Reinforcement fracture across the transverse crack was observed.

The failure of the slab was consistent with observations made by Foster (2006) and Bailey and Toh (2007b), with reinforcement fracture across the shorter span of the slab.
2.2.8 Test 8

The slab was reinforced with deformed reinforcement to a ratio of 0.4% by cross-sectional area. A load of 8.75kN/m², equivalent to the theoretical ultimate capacity, was placed on the slab. Fig. 2.25 shows the top and bottom views of the slab after the test.

Several cracks were seen across the short span of the slab, with a main transverse crack in the centre. During the test, however, yield lines were not clearly visible. On investigation of the bottom surface after the test, small discrete cracks were observed forming a yield line pattern (as shown in Fig. 2.25). There was no clear indication of
failure. The transverse tension cracks were limited to the central zone of the slab. A maximum vertical displacement of 52.2mm was recorded at a corresponding bottom surface temperature of 579°C at 226 minutes (Fig. 2.26).

![Graph showing vertical displacements in Test 8](image)

**Fig. 2.26: Vertical displacements in Test 8**

### 2.2.9 Test 9

The previous test was repeated with smooth reinforcement in Test 9. More discrete cracks were noticed in this test. The cracks followed a yield line pattern, and were observed during the test. Transverse cracks were also observed (Fig. 2.27), but they were not as clear as those of the less highly reinforced slabs. On the bottom surface of the slab, clearer patterns of the formation of yield lines were seen. There were also additional cracks running parallel to the edges of the slab, similar to the observations of Test 3.

A maximum vertical displacement of 67.2mm was recorded (Fig. 2.28). The discrete cracks and the lack of a clear through-depth transverse tension crack suggested that the slab behaviour had been altered by the higher reinforcement ratio. Subsequent slab tests were therefore devoted to the investigation of failure in relation to the reinforcement ratio.
2.2.10 Test 10

A concrete slab with 0.2% smooth reinforcement was used in Test 10. The slab was loaded to twice its theoretical capacity. Again, in this test, yield lines were observed before the transverse crack appeared. The transverse crack was seen at ambient-temperature. The transverse crack was observed to grow larger as vertical displacements increased. Fracture of reinforcement across this crack did not occur until the crack was between 5mm and 7mm wide. Fig. 2.29 shows the reinforcement intact during the test, although an integrity failure had occurred. At this stage the crack width
was about 3.5mm. The higher loading made crack patterns more visible during and after the test, as Fig. 2.30 shows.

![Fig. 2.29: Visible reinforcement intact during Test 10](image)

The maximum vertical deflection at ambient temperature was 25.4mm. This increased in the elevated-temperature phase to 85.5mm (Fig. 2.31). The test observation and crack patterns show that failure of the slab was by the fracture of reinforcement across the transverse crack.

![Fig. 2.30: Top and bottom slab faces in Test 10](image)

0.2% SM 0.9mm LR2 – Top face
0.2% SM 0.9mm LR2 – Bottom face
2.2.11 Test 11

Similar to Test 10, Test 11 had 0.2\% reinforcement ratio. Deformed wires of 0.9mm diameter were used. A test load of 8.64kN/m\(^2\) was applied to the slab. The formation of yield lines was recorded at about 7 minutes into the test. Transverse tension cracks were very small and did not open up with increasing vertical deflections. The prevailing crack patterns throughout the test were therefore at yield lines.

The ambient-temperature phase lasted for 60 minutes with a vertical deflection of 26mm. At about 1.5 hours into the test, the loading arrangement collapsed onto the slab, and therefore the test was stopped. The total vertical deflection recorded was 68.3mm. Reinforcement across the central yield line was observed to have fractured at the end of the test. Figs. 2.32 and 2.33 respectively show the two views of the slab and the recorded vertical displacements.
2.2.12 Test 12

The test was to investigate the failure mode of slabs with higher reinforcement ratios. A load of 17.66kN/m² was required for the test slab, which had been reinforced to a ratio of 0.4% using smooth wires of 0.9mm diameter. This represented twice the capacity of the slab. However, failure of the slab occurred before full application of the load. The test ended after 25 minutes as a significant vertical deflection of 76mm was reached. The load-central deflection plot recorded during the test is shown in Fig. 2.34.
The maximum applied load in the test was 7.72kN, equivalent to a uniform floor loading of 17.23kN/m² (including slab self weight and the weight of the loading equipment). No yield lines were seen in the slab either during the test or after the specimen had been removed from the furnace. The top and bottom views of the slab are shown in Fig. 2.35. Distributed cracks were seen on the bottom surface, with some cracks parallel to the edges. No transverse cracks observed during the test. The compressive crushing of the corners of the slab is further highlighted in Fig. 2.36.
2.2.13 Test 13

This was the final test, which was similar to Test 12. The slab was, however, reinforced with 0.4% deformed wire of 0.9mm diameter. The test lasted 20 minutes, at which time the load arrangement collapsed. However, a vertical deflection of 60.5mm was recorded. As Fig. 2.37 shows, no yield lines or transverse cracks were observed during the test, although yield line patterns were seen on the bottom face after the specimen had been removed from the furnace (Fig. 2.37). Crushing of concrete at the corners of the slab was seen with discrete, but small, cracks along the bottom surface. The failure of the slab resembled those seen in Tests 3 and 12. The load-deflection characteristic of Test 13 is shown in Fig. 2.38.
2.3 Discussion of Results

2.3.1 Effects of thermal gradient

The first two experiments were designed to test the effects of thermal gradient, acting alone, on the development of tensile membrane action. This was intended to investigate the differences in behaviour that had been observed by Foster (2006) in her ambient- and elevated-temperature experimental studies. Test slab 1, which was reinforced with 0.1% smooth wire of 0.71mm diameter, attained a central deflection of 14.6mm, which was of the order of the slab’s thickness (15mm), thereby clearly establishing the occurrence of tensile membrane action.

However, Test 2, reinforced with 0.1% deformed wires of the same diameter did not achieve the same result. The maximum central deflection was 6.07mm, which was less than half the thickness of the slab. It could not attain higher deflections, and the highest bottom surface temperature attained was 431°C. Therefore Vulcan was employed to investigate the development of the mechanism. The test temperatures at the bottom face, the top face and the reinforcement level, as well as the slab’s self weight were used as input for the analysis. The result, compared with the test deflection, is shown in Fig. 2.39.
Fig. 2.39 shows a very good comparison of the test behaviour with *Vulcan* analysis. *Vulcan* accurately predicts the history of the test slab, and therefore provides an accurate prediction of its deflections with temperature. Therefore, with adequate thermal exposure, the slab could easily have generated deflections of 12.8mm at a bottom surface temperature of 650°C. In Tests 1 and 2, corner cracks were observed, at fairly large distances from the corner supports, and transverse cracks were seen across the shorter spans of the slabs. The transverse cracks in Test 1 were only observed on the top surface of the slab, and not at the bottom. When a slab is exposed to a thermal gradient through its depth, and no transverse loading, the differential thermal expansion between the top and bottom surfaces causes it to bow downwards (towards the heat source). This differential thermal expansion causes the lower layers of the slab to be in compression while the upper layers are in tension. This could be the reason why cracks were seen only on the top face of Test slab 1. In Test 2, however, the transverse crack was observed on both faces.

### 2.3.2 Effects of reinforcement bond strength

As with previous tests using smooth and deformed reinforcement (Foster, 2006), the experimental investigation showed that slabs reinforced with smooth wires experienced larger deflections relative to similar slabs reinforced to the same degree, but with deformed wires. It was also observed that in general, more cracks were seen in
specimens with deformed reinforcement, as compared to those with smooth reinforcement.

This behaviour is due to the low bond strength between smooth reinforcement and concrete, and the higher ductility of the smooth wires. The relative movement between the materials allows the generation of larger deflections, while lower deflections are seen in the deformed-wire-reinforced specimens, with correspondingly larger cracks and fractures in reinforcement in certain cases. Figs. 2.40 and 2.41 show typical cases of the effects of bond strength on tensile membrane action. Fig. 2.40 shows a comparison between slabs reinforced to a 0.2% reinforcement ratio while Fig. 2.41 shows Tests 8 and 9, which had reinforcement ratios of 0.4%.

As tensile membrane action in thin vertically-supported concrete slabs increases with increasing deflections, the smooth-wire-reinforced slabs have higher capacities, and are less prone to integrity failures.

Fig. 2.40: 0.2% reinforcement ratio comparison
2.3.3 Effects of reinforcement ratios

Previous researchers have found that there is an implicit assumption in the Bailey-BRE method that slab capacities increase with increasing reinforcement size. The recent improvements in the method, however, address this issue; a failure criterion has been incorporated into the method so that compressive failures (which occur as the result of high reinforcement) can be accounted for. The Bailey-BRE method determines slab resistance in fire by calculating the additional contribution of membrane action to the traditional yield-line capacity by calculating enhancement factors, which are dependent on the magnitude of the vertical deflection and the increased bending resistance in compression regions of the slab.

Fig. 2.42 shows the deflections of Tests 4, 5 and 7, which were all reinforced with deformed wire, to ratios of 0.2%, 0.1% and 0.4% respectively. Since loads equivalent to each slab’s capacity were placed on the specimens, the deflections show increasing slab capacity with increasing reinforcement ratio. This increase is, however, not proportional to the increase in reinforcement ratio, as described below. The deflections of Tests 5, 4 and 7 at a bottom surface temperature of 550°C are 17.9mm, 33.0mm and 43.4mm respectively.
Fig. 2.42: Vertical displacements of Tests 5, 4 and 7

Using the Bailey-BRE method (Bailey and Toh, 2007a), the enhancement factors at the deflections mentioned above are calculated as 1.452, 1.848 and 2.145 for 0.1%, 0.2% and 0.4% reinforcement ratios, based on yield-line capacities of 2.13kN/m², 3.71kN/m² and 7.08kN/m², respectively. The factors are plotted in Fig. 2.43, and the gradients are determined as 3.96 between 0.1% and 0.2%, and 1.49 between 0.2% and 0.4%.

Fig. 2.43: Enhancement factors from Tests 5, 4 and 7
It is obvious therefore, from the graph, that as higher reinforcement ratios are encountered, the increase in overall slab capacity is due to its higher flexural capacity rather than the enhancement obtained through the larger deflection.

Due to the simple nature of the Bailey-BRE method, it is an ideal tool for the prediction of concrete slab behaviour at large deflections. By employing rigid-plastic behaviour in its calculations, the method produces an upper limit to slab behaviour in fire.

Fig. 2.44 shows Vulcan and Bailey-BRE predictions of Test 5 compared with the actual test deflection. In Test 5 the slab was reinforced with deformed 0.9mm diameter wires to a ratio of 0.1%. The Bailey deflection here is termed the ‘Required Bailey deflection’. It is supposed to be a conservative estimate of the slab’s deflection. It is observed that the test deflections and the Vulcan prediction all lie above this limit, therefore confirming the test observation of no failure of Test 5.

However, when a similar prediction is made with respect to Test 7, where a reinforcement ratio of 0.4% was used, it is observed that, the Bailey prediction ceases to be conservative after a temperature of about 60°C when the vertical deflection is 9.7mm (Fig. 2.45). At higher slab temperatures, the prediction approaches the actual test deflection as reinforcement temperatures increase considerably. It is noted that the test deflections exceed the required Bailey-BRE deflection, with no observed
failures. The *Vulcan* predictions are seen to represent the test behaviour closely in both tests.

![Graph](image)

**Fig. 2.45: Bailey-BRE and *Vulcan* comparisons in Test 7**

From Figs. 2.44 and 2.45, it can be concluded that the Bailey-BRE method does not accurately predict the behaviour of highly reinforced slabs in fire. The difference in the two results is primarily due to the higher slab bending resistance in Test 7 (because of the higher reinforcement ratio) and the lack of an adequate treatment of thermal actions in the method, especially in the early stages of the analysis when reinforcement temperatures are below 400°C. The prediction is seen to be adequate towards the end of the test, when the reinforcement behaviour is greatly influenced by the higher temperatures.

### 2.3.4 Slab failure modes

In general, two failure modes were observed in the experimental program. Slabs either failed by the fracture of reinforcement at the centre of the slab across the transverse crack, or by the crushing of concrete at the corners of the slab. Slabs loaded to the theoretical ultimate capacity did not fail or show clear crack patterns. They however showed the formation of yield lines in cases of either low or high reinforcement ratios. With lower ratios, the transverse crack was seen, while with higher ratios, crack patterns were often formed by many distributed cracks, which were aligned in yield line patterns but with no clear transverse crack (Fig. 2.46).
To monitor the failure modes of slabs with various reinforcement ratios therefore, twice the theoretical ultimate load was placed on the slab. Fig. 2.47 shows the crack patterns of the two failure modes.

The results show that slabs reinforced to lower reinforcement ratios failed by the fracture of reinforcement across the transverse crack, while those with higher reinforcement ratios failed by compressive crushing of concrete at the slab corners.

None of the compressive failure modes of the highly reinforced slabs were seen at elevated temperatures; they all occurred at ambient temperature, while the slabs were still being loaded.
Figs. 2.48 and 2.49 show the applied load – central vertical displacement plots of Test 12 and Test 3 respectively, which have been compared with corresponding *Vulcan* predictions. The graphs show good correlation between *Vulcan* and the tests. To assess the failure mechanism of the highly reinforced slabs, a *Vulcan* analysis of Test 3 examines the development of membrane tractions and crack patterns in the slab. The results are shown in the subsequent figures.

**Fig. 2.48: *Vulcan* prediction of the load-deflection behaviour of Test 12**

**Fig. 2.49: *Vulcan* prediction of the load-deflection behaviour of Test 3**
Figs. 2.50, 2.51 and 2.52 show membrane traction plots of Test 3 in three stages. Fig. 2.50 shows the incidence of tensile membrane action, at a load of 2.7kN. In the figure, tensile tractions (in the middle of the slab) are shown in red, with compressive tractions shown in blue. In the initial stages of membrane action, the tensile tractions are limited to the centre of the slab at relatively low deflections (6mm).

![Fig. 2.50: Test 3 membrane tractions - Stage 1](image)

Fig. 2.50 shows membrane tractions at Stage 2. It is observed that the area of tensile tractions increases, as deflections increase. This constrains the compressive tractions to a smaller area of the slab, thereby increasing the compressive strains in these areas. As observed from Fig. 2.52, at failure, the higher loading generates tensile tractions which occupy most of the slab area. The high compressive strains in the slab, especially at the corners therefore cause failure of the slab.

![Fig. 2.51: Test 3 membrane tractions - Stage 2](image)

The *Vulcan* analysis indicates that at a load of 13.65kN/m² (equivalent to an applied load of 6.04kN) compressive failure of the slab elements occurs near the corners. The
top layer compressive stress in these slab elements is about 33.53N/mm² (equivalent to a cube strength of 42.65N/mm²), which is greater than the compressive strength of the slab. This explains the crushing of concrete that was observed in the test, as represented in the *Vulcan* crack patterns (Fig. 2.53).

![Fig. 2.52: Test 3 membrane tractions - Stage 3](image)

![Fig. 2.53: Test 3 crack patterns - topmost layer](image)

### 2.4 Summary

Thirteen tests were conducted on model-scale concrete slabs at elevated temperatures. The experiments examined the effects of thermal gradients and bond strengths on the development of tensile membrane action in thin slabs. The effects of reinforcement ratios on slab behaviour and failure at elevated temperatures have also been investigated.
The tests show that thermal gradients, acting alone, can induce a considerable amount of tensile membrane action. The differential thermal expansion between the bottom and top surfaces of the slab induces large deflections and the formation of the transverse tension crack.

The study also showed that slabs reinforced with smooth wires, which had higher ductility but lower bonds with concrete exhibited higher slab capacities than equivalent slabs with deformed wires. This highlighted the contribution of bond-slip to tensile membrane action.

The experiments, conducted with ultimate load ratios of 1.0 and 2.0, showed that slab behaviour at elevated temperatures was influenced by reinforcement ratios. Low reinforcement ratios generated clear transverse cracks with subsequent yield-lines towards failure, while highly reinforced slabs showed discrete cracks aligned in the shape of yield lines, with no clear transverse cracks. It was also observed that highly reinforced slabs failed by compressive crushing of concrete at their corners. However, this behaviour was only seen at ambient temperature.

Comparisons of the tests with the numerical software Vulcan and the revised Bailey-BRE method showed that the numerical software generally had good correlation with the test results, while the Bailey-BRE prediction only suited lightly-reinforced slabs.
3 Thermal Gradients

A brief review of some of the existing methods for composite slab analyses in fire has been given in Chapter 1. These include the Bailey-BRE method and the approach developed by the University of Edinburgh for laterally restrained slabs. The Bailey-BRE method accounts for thermal effects in slabs, by employing reduced capacities of concrete and reinforcement at elevated temperatures and by limiting the vertical deflections on the assumption of a linear thermal gradient through the depth of the slab. The method developed by the University of Edinburgh accounts for thermal effects in slabs by generating a vertical deflected shape of the slab based on a thermal gradient through the depth of the slab. However, it relies on the anchorage of reinforcement along slab panel boundaries for the development of tensile membrane action. Tests by Foster (2006), and those described in the preceding chapter, show that considerable tensile membrane action develops in horizontally-unrestrained simply-supported slabs subject to thermal gradients.

This chapter therefore explores the possibility of quantifying tensile membrane capacity in horizontally-unrestrained simply-supported slabs by taking account of thermal exposure only. The study is seen as an initial step to the development of a simple method of calculating the capacity of composite slabs in fire. The study uses classical large-deflection slab theory and compares its results with finite element analyses. Parts of the results of this chapter were presented at the 4th Structures in Fire Workshop in Aveiro, Portugal in 2006 (Abu et al., 2006).

3.1 Thermal actions and slab behaviour
As the Bailey-BRE method was developed from ambient-temperature considerations, it calculates the membrane enhancement of slabs by using the reduced strengths of concrete and reinforcement at various temperatures. Its calculations, however, are based on the formation of the small-deflection yield-line mechanism before the occurrence of tensile membrane action; a theory which is not valid at elevated temperatures, as the tests discussed in the preceding two chapters show. The method also assumes that slabs are treated as simply supported, because of the large hogging moments that may be generated along slab panel boundaries in fire. In the analyses of composite slabs for tensile membrane action equivalent flat slabs are used, due to the
high temperatures sustained at the soffit of the slab and the observed separation of the concrete slab from the steel decking in fire (Bailey, 2000).

A new design method, developed from classical large-deflection theory by Cameron and Usmani (2005a; 2005b), proposes that composite slabs should be analysed through a three-step process. The analytical method generates a temperature distribution through the slab; consisting of a mean temperature increase and a thermal gradient, then determines the vertical deflection and stress-strain distribution due to the thermally induced strains and, using an energy method, calculates the membrane capacity of the slab, under the assumption that that the tensile forces developed in the slab can only be balanced by the provision of anchorage along the slab boundary as most of the slab bending capacity will be lost at elevated temperatures. It also assumes that this horizontal anchorage will be provided by the adjacent slabs where an interior slab is concerned, but proposes that the design of composite beams on the edges of buildings, and their connections, should account for the required lateral restraint to the slabs in addition to the vertical restraint provided.

As described previously, tests by Foster (2006) on small-scale horizontally-unrestrained simply-supported flat concrete slabs at the University of Sheffield have shown that the mode of failure of concrete slabs in fire differs from what is observed at ambient temperature. At elevated temperatures, thermal bowing of the slab induces double-curvature bending which generates full-depth tensile cracking failure across the shorter span of the slab, which may lead to an eventual yield-line type of failure mechanism (Foster et al., 2005). The observations and the magnitudes of vertical deflections reached in these small-scale tests, and the authors own small-scale tests, modelled on Foster’s, show that thermal gradients acting alone through the depth of the slab can cause significant amounts of tensile membrane stress in axially-unrestrained simply-supported slabs. Contrary to the suggestion by Cameron and Usmani (2005a; 2005b), it is not necessary to provide horizontal edge restraint to sustain this load-carrying mechanism at elevated temperatures.

The research reported in this chapter investigates the effects of these thermal gradients on the development of tensile membrane action, and the associated stresses, using the variational Rayleigh-Ritz Method. The development of tensile membrane action at ambient and elevated temperatures is considered, and comparisons are made with the finite element package *Vulcan* (Huang et al., 2003a; 2003b).
3.2 Large-deflection plate theory

The most common dimensions of slabs fall into a category of plates known as medium-thickness plates. These are plates which have moderate thicknesses, such that their out-of-plane shear deformations and in-plane membrane forces can be ignored (Park and Gamble, 2000) at small deflections. Classical plate theory (Timoshenko and Woinowsky-Krieger, 1959) provides various solutions to plate problems for this class, on the assumption that these plates are made of a linearly elastic material. The basic assumptions are:

1. the deflection of the mid-surface is small compared with the thickness of the plate;
2. the mid-plane remains unstrained subsequent to bending;
3. lines initially normal to the mid-surface remain normal to that surface after bending;
4. stresses normal to the mid-plane are small compared to those in the plane of the plate, and are therefore ignored.

By considering equilibrium conditions and strain compatibility, the established governing equation of medium-thickness plates is:

\[
\frac{\partial^2 w}{\partial x^2} + 2 \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 w}{\partial y^2} = \frac{q}{D}
\]

(3.1)

where \( w \) is the deflection of the slab, \( q \) is the load per unit area and \( D \) is the rigidity of the plate. The set of strains at a depth \( z \) through the cross-section of the slab is given by:

\[
\varepsilon_x = -z \frac{\partial^2 w}{\partial x^2}, \quad \varepsilon_y = -z \frac{\partial^2 w}{\partial y^2}, \quad \gamma_{xy} = -2z \frac{\partial^2 w}{\partial x \partial y}
\]

(3.2)

In fire, however, the combination of the increased loading on composite slabs and material degradation requires the examination of large deflections of plates. At large deflections, exceed the thickness of the slab, strains are generated in the mid-plane, thereby rendering the first two assumptions of the medium-thickness plate theory invalid. Therefore, expressions for strain are modified accordingly, to account for the stretching of the mid-plane of the plate and the nonlinear effect of the transverse loading. These are shown by the respective first and second terms in the large-deflection strain-displacement relationship (Equation 3.3). In Equation 3.3, the in-plane displacements in the \( x \) and \( y \) directions are denoted by \( u \) and \( v \), respectively.
\[ \varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 - z \frac{\partial^2 w}{\partial x^2} \]

\[ \varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 - z \frac{\partial^2 w}{\partial y^2} \]

\[ \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = 2z \frac{\partial^2 w}{\partial x \partial y} \] (3.3)

The governing equations of large-deflection theory are therefore (Timoshenko and Woinowsky-Krieger, 1959):

\[ \frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E \left[ \frac{\partial^2 w}{\partial x^2 \partial y} - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] \] (3.4)

\[ \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = h \left( \frac{q}{h} + \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \right) \] (3.5)

In the above equations \( F \) is a stress function such that:

\[ N_x = h \frac{\partial^2 F}{\partial y^2}, \quad N_y = h \frac{\partial^2 F}{\partial x^2}, \quad N_{xy} = -h \frac{\partial^2 F}{\partial x \partial y} \] (3.6)

and \( N_x, \ N_y \) and \( N_{xy} \) are the forces per unit length in the \( x, y \) and \( xy \) directions respectively. For a simply-supported plate whose origin of co-ordinates is at a corner, as in Fig. 3.1, exact solutions to the governing equations of large deflection theory can be obtained if the following functions are defined as:

\[ w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} w_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \] (3.7)

\[ q = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \] (3.8)

\[ F = \frac{P_x y^2}{2bh} + \frac{P_y x^2}{2ah} + \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} f_{mn} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} \] (3.9)

Approximate solutions for the plate load-deflection behaviour can be obtained if energy methods are employed. One such energy method is the variational Rayleigh-Ritz Method. This method requires that the mechanical strain energy of the plate, considering both stretching and bending, is obtained, and the amplitudes of any shape functions used in the generation of the strain and potential energies are determined by
minimising the total potential energy. The determined coefficients can then be used to approximate the deflected shapes and stresses in the plate.

\[ V = \frac{E}{2(1-v^2)} \int \left[ \varepsilon_x^2 + \varepsilon_y^2 + 2v\varepsilon_x\varepsilon_y + \frac{1-v}{2}\gamma_{xy}^2 \right] dx dy dz \] (3.10)

\[ \varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 - z \frac{\partial^2 w}{\partial x^2} \]

\[ \varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 - z \frac{\partial^2 w}{\partial y^2} \] (3.11)

\[ \gamma_{xy} = \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} - 2z \frac{\partial^2 w}{\partial x \partial y} \]

To use the Rayleigh-Ritz Method functional expressions are defined for the displacements in the \( x \), \( y \) and \( z \) directions such that these expressions satisfy the geometric and natural boundary conditions.

### 3.2.1 Boundary conditions

The slab is oriented as shown in Fig. 3.2. Given the apparent symmetry of the problem, a quarter of the slab is analysed. The geometric and natural boundary conditions are given in Equation 3.12.

\[ \text{At } x = \pm \frac{a}{2}, \quad N_x = 0, \quad M_x = 0, \quad w = 0, \quad \frac{\partial^2 w}{\partial x^2} = 0, \quad \frac{\partial^2 w}{\partial y^2} = 0 \]

\[ \text{At } y = \pm \frac{b}{2}, \quad N_y = 0, \quad M_y = 0, \quad w = 0, \quad \frac{\partial^2 w}{\partial x^2} = 0, \quad \frac{\partial^2 w}{\partial y^2} = 0 \] (3.12)
3.3 Solution of the large-deflection problem

3.3.1 Research on Solutions of Large-Deflection Plate Problems
Attempts have been made by a number of researchers to approximate vertical deflections and stresses in thin plates under large deflections, using the principle of minimum potential energy. Berger (1955) proposed a strain energy equation that ignored the second invariant of the mid-surface strains. However, subsequent research established the ineffectiveness of this principle for plates with movable boundaries (Nowinski and Ohnabe, 1958). Banerjee and Datta (1981) proposed a method which linearised the total potential energy with an expression, in terms of a factor $\lambda$ and the vertical deflection, which gave good results if the correct factor was chosen. However, there could be no physical justification for the selection of specific values for $\lambda$. Boresi and Turner (1983) proceeded by maintaining the full non-linear energy equations, and defining functional expressions for in-plane strains in the $x$ and $y$ directions, and the vertical deflection. Odd-numbered double-Fourier series expansions were used for the strains and the vertical deflections (Equation 3.13).

$$
\varepsilon_x = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b}, \quad \text{where } m,n = 1,3,5,\ldots
$$

$$
\varepsilon_y = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \cos \frac{n \pi y}{b} \cos \frac{m \pi x}{a}, \quad \text{where } m,n = 1,3,5,\ldots
$$

$$
w = W_{nn} \cos \frac{\pi x}{a} \cos \frac{\pi y}{b}
$$

(3.13)
3.3.2 Adopted Solution

For large deflection of plates, the mid-surface strains depend on the stretching of the mid-surface and the contribution of vertical deflection. Preliminary finite element analysis and observations from the tests discussed earlier showed that, for simply supported slabs subjected to large deflections, the edges (including corners) are pulled-in. However, the expressions proposed by Boresi and Turner (1983) artificially keep the corners fixed in position, preventing the slab from attaining appreciable magnitudes of displacement. In practice, this will indicate that failure of slabs in tensile membrane action will not affect the columns at the corners of the slab panel. The Boresi and Turner expressions (Equation 3.13) are therefore modified to suit the observed behaviour. In this investigation the full Fourier series is therefore used.

3.4 Model derivation

In Section 3.2, it was mentioned that the use of the Rayleigh-Ritz method requires the definition of displacement equations in the three primary directions; strains and stresses are derived after the amplitudes of the shape functions have been obtained. However, due to the coupled highly nonlinear equations, the membrane strains are defined first, following Boresi and Turner (1983). The in-plane displacements are then derived. For a horizontally unrestrained simply-supported slab, with its origin at its centre, as shown in Fig. 3.2, the total in-plane strains at the mid-surface in the $x$ and $y$ directions and the vertical displacement are defined as:

$$
\varepsilon_x = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b}, \text{ where } m, n = 1, 2, 3, \ldots
$$

(3.14)

$$
\varepsilon_y = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \cos \frac{m \pi y}{b} \cos \frac{n \pi x}{a}, \text{ where } m, n = 1, 2, 3, \ldots
$$

(3.15)

$$
w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} W_{mn} \cos \frac{m \pi x}{a} \cos \frac{n \pi y}{b}, \text{ where } m, n = 1, 3, 5, \ldots
$$

(3.16)

It is observed from the strain expressions that if $m$ and $n$ include even numbers, the strains at the edges of the slab ($x = a/2$ and $y = b/2$) are non-zero. This contradicts the basic principles of equilibrium, as the membrane tractions along the edges are then non-zero (see Equation 3.17).

$$
N_x = \frac{Eh}{1 - \nu^2} (\varepsilon_x + \nu \varepsilon_y), \quad N_y = \frac{Eh}{1 - \nu^2} (\varepsilon_y + \nu \varepsilon_x)
$$

(3.17)
Using only odd-numbered terms, as suggested by Boresi and Turner (1983) however, stops in-plane displacements, as will be demonstrated later. The eventual adoption of the expressions above is based on the nature of the Rayleigh-Ritz solution, which, by minimising the potential energy, assigns higher amplitudes to the mode-shapes that aid a particular solution while reducing the amplitudes of those mode shapes that are detrimental. It should be noted that the expressions satisfy all other boundary conditions.

A linear thermal gradient is assumed through the depth of the slab, with \( T_1, T_2 \) and \( T_m \) as the top surface, bottom surface and mid-surface temperatures, respectively as shown in Fig. 3.3. \( T_0 \) is the ambient temperature with the thickness of the slab denoted as \( h \).

![Fig. 3.3: Thermal gradient through the depth of the slab](image)

From the diagram above, the thermal strain at any depth \( z \), at elevated temperature, is defined as:

\[
\varepsilon_T = \alpha \Delta T
\]

where

\[
\Delta T = (T_m - T_o) + z \frac{T_2 - T_1}{h}
\]

\[
T_m = \frac{T_1 + T_2}{2}
\]  

The stress-related strains (mechanical strains) at any point in the slab can therefore be obtained by the respective strains in the \( x \), \( y \) and \( xy \) directions:

\[
\varepsilon_{x(mech)} = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 - \alpha \Delta T - z \frac{\partial^2 w}{\partial x^2}
\]
\[ \varepsilon_{y(mech)} = \frac{\partial y}{\partial y} + \frac{1}{2} \left( \frac{\partial w}{\partial y} \right)^2 - \alpha \Delta T - \frac{1}{2} \frac{\partial^2 w}{\partial y^2} \]  

(3.20)

\[ \gamma_{xy(mech)} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} - 2 \varepsilon \frac{\partial^2 w}{\partial x \partial y} \]  

(3.21)

It is observed from Equation 3.21 that the in-plane shear strain requires the definition of the displacement functions in the \( x \) and \( y \) directions. These are obtained by re-arranging Equations 3.19 and 3.20, and integrating the respective functions with respect to \( x \) and \( y \).

At the mid-surface, \( z = 0 \), thus Equation 3.19 can be re-written as:

\[ \varepsilon_{x(mech)} = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 - \alpha \left( T_m - T_0 \right) \]  

(3.22)

The \( x \)-direction displacement, at a point in the mid-surface, can be found by:

\[ u = \int \left[ \varepsilon_{x(mech)} - \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 + \alpha \left( T_m - T_n \right) \right] dx \]  

(3.23)

From the expression for the out-of-plane displacement \( w \) (Equation 3.16),

\[ \frac{1}{2} \left( \frac{\partial w}{\partial x} \right)^2 = \sum_{m=odd}^{\infty} \sum_{n=odd}^{\infty} m^2 \pi^2 \left( 1 - \cos \frac{2m \pi x}{a} \right) \left( 1 + \cos \frac{2n \pi y}{b} \right) \]  

(3.24)

Therefore \( u \) becomes:

\[ u = \frac{a}{\pi} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{m} A_{mn} \sin \frac{m \pi x}{a} \cos \frac{n \pi y}{b} \]

\[ - \frac{\pi^2}{8a^2} \sum_{m=odd}^{\infty} \sum_{n=odd}^{\infty} m^2 W_{mn} \left( x - \frac{a}{2m \pi} \sin \frac{2m \pi x}{a} \right) \left( 1 + \cos \frac{2n \pi y}{b} \right) + \alpha \left( T_m - T_n \right) x \]  

(3.25)

Similarly, \( v \) is:

\[ v = \frac{b}{\pi} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{1}{n} B_{mn} \sin \frac{n \pi y}{b} \cos \frac{m \pi x}{a} \]

\[ - \frac{\pi^2}{8b^2} \sum_{m=odd}^{\infty} \sum_{n=odd}^{\infty} n^2 W_{mn} \left( y - \frac{b}{2n \pi} \sin \frac{2n \pi y}{b} \right) \left( 1 + \cos \frac{2m \pi x}{a} \right) + \alpha \left( T_m - T_n \right) y \]  

(3.26)

It is observed from Equations 3.25 and 3.26 that at the corners (\( x = a/2 \) and \( y = b/2 \)), in-plane displacements occur, because of the even numbered terms of the in-plane expressions, while Boresi and Turner’s expressions (Equation 3.13) restrain the corners, so that displacements are zero.
Substitutions of the respective first partial derivatives of Equations 3.25 and 3.26 with respect to y and x, and the partial derivatives of Equation 3.16 with respect to x and y, into Equation 3.21 yields the in-plane shear strain \( \gamma_{xy} \).

After all the strains have been obtained, the total strain energy of the plate is obtained through:

\[
V = \frac{E h}{2(1-v^2)} \left[ \int \left( \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial y} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right) \right] dx dy
+ \frac{E h}{2(1-v^2)} \left[ \int \left( \frac{1}{4} \left( \left( \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial w}{\partial y} \right)^2 \right) \right) + 2v \left( \frac{\partial u}{\partial x} \frac{\partial w}{\partial y} + \frac{\partial v}{\partial y} \frac{\partial w}{\partial x} \right) \right] dx dy
+ \frac{E h}{2(1-v^2)} \left[ \int \left( \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial x} \right)^2 \right) + 2v \left( \frac{\partial u}{\partial x} \frac{\partial w}{\partial y} + \frac{\partial v}{\partial y} \frac{\partial w}{\partial x} \right) \right] dx dy
+ \frac{E h^3}{24(1-v^2)} \left[ \int \left( \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) - 2(1-v) \left( \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left( \frac{\partial^2 w}{\partial x \partial y} \right)^2 \right) \right) \right] dx dy
+ \frac{E h^3}{24(1-v^2)} \left[ \int \left( 2(1+v) \left( \frac{T_2 - T_1}{h} \right) \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + 2\alpha^2 (1+v) \left( \frac{T_2 - T_1}{h} \right)^2 \right) \right] dx dy
\] (3.27)

Potential energy is calculated from the difference between the strain energy and the work done by the applied load, which is given by:

\[
\text{Work Done} = \int \int w. q dx dy
\] (3.28)

The amplitudes of the various mode shapes are then obtained by the solution of the simultaneous equations that arise by minimising the potential and strain energies (Equations 3.29 and 3.30, respectively).

\[
\frac{\partial}{\partial W_{mn}} \left( V - \int \int w. q dx dy \right) = 0
\] (3.29)

\[
\frac{\partial V}{\partial A_{mn}} = \frac{\partial V}{\partial B_{mn}} = 0
\] (3.30)
Due to the symbolic nature of the calculations, the software MAPLE 9.5 was used to solve for the amplitudes of the modal shapes. Displacements and stress-related strains are then determined:

\[
\varepsilon_{x(mech)} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} - \alpha(T_m - T_0) - \alpha z \frac{T_z - T_1}{h} - z \frac{\partial^2 w}{\partial x^2} \tag{3.31}
\]

\[
\varepsilon_{y(mech)} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} \cos \frac{n\pi y}{b} \cos \frac{m\pi x}{a} - \alpha(T_m - T_0) - \alpha z \frac{T_z - T_1}{h} - z \frac{\partial^2 w}{\partial y^2} \tag{3.32}
\]

\[
\gamma_{xy(mech)} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} - 2z \frac{\partial^2 w}{\partial x \partial y} \tag{3.33}
\]

The stresses at any point and membrane tractions in the slab are calculated:

\[
\sigma_x = \frac{E}{1 - \nu^2} \left( \varepsilon_{x(mech)} + \nu \varepsilon_{y(mech)} \right)
\]

\[
\sigma_y = \frac{E}{1 - \nu^2} \left( \varepsilon_{y(mech)} + \nu \varepsilon_{x(mech)} \right) \tag{3.34}
\]

\[
\tau_{xy} = G \gamma_{xy(mech)}
\]

\[
N_x = \frac{Eh}{1 - \nu^2} \left( \varepsilon_{x(mech)} + \nu \varepsilon_{y(mech)} \right)
\]

\[
N_y = \frac{Eh}{1 - \nu^2} \left( \varepsilon_{y(mech)} + \nu \varepsilon_{x(mech)} \right) \tag{3.35}
\]

### 3.5 Model validation

Tensile membrane action is deemed to have developed in a thin plate when vertical displacements are of the order of the thickness of the plate, regardless of the type of action imposed on the plate. The model is therefore validated at ambient temperature to ensure that deflections and stresses at large deflections are being obtained accurately.

#### 3.5.1 Ambient Temperature Validation

A 100kN/m² load is imposed on a square flat slab of 5m side length and thickness of 100mm. The plate has modulus of elasticity of 18000N/mm² and Poisson’s ratio of 0.25. Comparisons of the behaviour of the analytical model were made with *Vulcan*, the University of Sheffield specialist finite element code (Huang *et al.*, 2003a; 2003b), using a linear elastic material of the same properties and 100mm thick slab, divided into 10 equal layers. The suitability of verifying the analytical model with *Vulcan* was
confirmed by modelling the same slab in ABAQUS. The results of the out-of-plane and in-plane displacements are shown in Figs. 3.5 and 3.6. In the comparisons that follow, the analytical model is represented by the solid line. *Vulcan* results are shown as dashed lines while those obtained with ABAQUS are as dotted lines.

For completeness, the slab behaviour was monitored along three lines across the slab; these were along the slab centre-line, the slab edge and mid-way between these two locations, designated by $y = 0$, $y = b/2$ and $y = b/4$ respectively, as shown in Fig. 3.4. Accordingly, the ambient temperature comparisons and all subsequent evaluations of the analytical model use the colour code shown in Fig. 3.4 to distinguish the locations of the results being discussed. In all cases, the variation of a particular parameter is determined along the length of the slab quarter section.

The analytical vertical deflection and the numerical results from *Vulcan* and ABAQUS are in very good agreement. Fig. 3.5 shows the ambient-temperature central vertical deflection as 124mm, which is greater than the thickness of the slab, confirming the tensile membrane mechanism. The horizontal displacements (Fig. 3.6) emphasize the existence of a tensile area with a surrounding ring of compression. The central portion of the plate is stretched, but the edges and corners are pulled-in towards the centre of the slab, creating the ring of compression.
A plot of membrane force per unit length across the span is shown in Fig. 3.7. Tensile forces are observed in the middle of the slab, and these decrease in magnitude and eventually become compressive forces towards the edge. Across the boundary, membrane tractions are zero in accordance with the local equilibrium conditions.
(Equation 3.12). There is a deviation from the expected traction result as the corner is approached.

![Graph showing ambient-temperature membrane tractions across span](image)

**Fig. 3.7: Ambient-temperature membrane tractions across span**

Membrane tractions and stresses of the *Vulcan* slab elements are obtained at Gauss points. Stresses are calculated at the 9 Gaussian integration points at each layer level. Membrane tractions (forces per unit length) are an integration of stresses through the depth of a slab at a particular Gauss point. Along the length of the isoparametric slab element, the Gaussian integration points are at $-\sqrt{3/5}$, 0 and $+\sqrt{3/5}$. Since only a quarter of the entire slab was modelled the membrane tractions and stresses nearest to the locations labelled $y = 0$, $y = b/4$ and $y = b/2$ were recorded, and compared with the results of the analytical model at similar locations. The slab element size used in the analysis was 250mm x 250mm.

Figs. 3.8 and 3.9 confirm the presence of tensile and compressive stresses (in the bottom and top layers respectively) of a plate loaded at ambient temperature. They also show the compressive stresses that develop along the mid-section of the edge due to its inward movement. The stresses reduce in magnitude along the edge, towards the corners.
Comparisons of the through-depth stresses at various locations in the slab are plotted in Figs. 3.10 to 3.12. As has been observed with the other parameters of the analytical model, there are good comparisons, especially in the middle region of the slab. The comparison is not as good in the region of the edge or corners of the element, as is evidenced in Fig. 3.12.
Chapter 3: Thermal Gradients

Fig. 3.10: Ambient-temperature through-depth stress at \( x = 0 \)

Fig. 3.11: Ambient-temperature through-depth stress at \( x = a/4 \)
For the purposes of this study on tensile membrane action, the interest is on failure of isolated slabs at very large deflections, unlike the earlier studies by Cameron and Usmani (2005a; 2005b). Failure, as observed from the earlier experiments, is either by tensile fracture of reinforcement in the middle of the slab, or by compressive crushing of concrete at its corners. So far no elevated-temperature analyses have established compressive failure of slabs in tensile membrane action (Bailey and Toh, 2007a; 2007b). The good analytical prediction, especially in the middle region of the slab is a good basis for its extension to the elevated-temperature phase.

### 3.5.2 Elevated-temperature Validation

At elevated temperatures the thermal strain $\varepsilon_i = \alpha \Delta T$ (where $\alpha$ is the coefficient of thermal expansion and $\Delta T = T - T_0$ is the change in temperature at the depth being considered) influences the mechanical strains developed in the plate. The validation is performed with the 100mm thick 5m x 5m square slab, and then verified with a 9m x 6m rectangular slab of the same thickness. In the analyses $\alpha$ is kept constant at $10 \times 10^{-6}$/°C.

As the aim is to investigate the effects of thermal gradients only, steady state conditions are used and a version of Vulcan which maintains room-temperature material properties is used for the comparison. The Young’s modulus and Poisson’s ratio of the material are therefore kept constant at 18000N/mm$^2$ and 0.25 in all layers, regardless of the imposed temperature gradient. No loads are imposed on the plate for
the elevated-temperature analyses. The analyses were conducted using a linear thermal gradient of 7° C/mm. This was achieved by maintaining the top of the slab at 20° C and increasing the temperature at the bottom surface to 720° C. The displacements, strains and stresses were determined as outlined in Section 3.4. The results for the square slab are discussed first, followed by the rectangular slab.

3.5.2.1 5m x 5m Square slab

Fig. 3.13 shows the elevated-temperature vertical displacements of the square slab. The graph shows results with the analytical Rayleigh-Ritz approach, Vulcan and ABAQUS in solid, dashed and dotted lines respectively. The close comparison between the results is notable. The central vertical displacement exceeds the 100mm thickness of the slab in all three models, indicating the presence of tensile membrane action. This observation is confirmed by the in-plane displacements of the slab shown in Fig. 3.14.

![Fig. 3.13: Elevated-temperature square slab vertical displacements](image)

In comparison to Fig. 3.6, the overall displacements in Fig. 3.14 are positive. This is due to the effect of thermal expansion, by virtue of the 350° C difference between the average temperature at the mid-surface level and the ambient temperature of 20° C. It is observed from the figure, however, that the central portions of the slab displace outwards because of the central tensile area, while the outer regions contract. The overall displacements therefore depend on the relative magnitudes of the mechanical and thermal strains.
Fig. 3.14: Elevated-temperature square slab in-plane displacements

Elevated-temperature membrane tractions across the span of the square slab are plotted in Fig. 3.15. The good agreement, between the Vulcan and Rayleigh-Ritz models, shows the analytical model’s prediction of a central tensile area and a peripheral compressive ring. The observation of the displacements and the membrane tractions at elevated temperature confirm that thermal gradients can generate considerable tensile membrane action, even when acting alone.

Fig. 3.15: Elevated-temperature square slab membrane tractions
Stress distributions in the lowest and topmost layers of the model and their comparisons with *Vulcan* are shown in Figs. 3.16 and 3.17. From these figures, it can be seen that the bottom of the slab is in compression, while the top is in tension. This is due to the differential thermal expansion through the depth of the slab. Higher temperatures at the base of the slab attempt to expand the lower layers, which are being restrained by the cooler upper layers; this induces compression, while at the top the stretching of the colder layers by expanding lower layers creates tension.

**Fig. 3.16: Elevated-temperature square slab bottom layer stress**

**Fig. 3.17: Elevated-temperature square slab top layer stress**
From Figs. 3.16 and 3.17, the analytical model accurately predicts the behaviour of the slab in its central portions, while deviations are seen towards the edges. An investigation of the stress distribution through the depth of the slab confirms this. An examination of Figs. 3.18, 3.19 and 3.20 show the divergence of the good comparison between \textit{Vulcan} and the Rayleigh-Ritz approach in moving from the centre \((x = 0, y = 0)\) to the corner \((x = a/2, y = b/2)\). A possible explanation of this phenomenon will be provided after the examination of the rectangular slab.
3.5.2.2 9m x 6m Rectangular slab

By exposing the slab to a bottom surface temperature of 720°C while keeping the top at 20°C, a mid-surface temperature of 370°C was obtained. The same linear-elastic material was used. Following the quarter-slab model of Fig. 3.2, the longer span length was represented by $a$, with $b$ as the shorter span length. The central vertical displacement of the slab was more than twice the thickness of the slab (Fig. 3.21).
The large vertical displacement clearly indicates the occurrence of tensile membrane action. Figs. 3.22 and 3.23 show the in-plane displacements of the slab along the respective long and short spans. Similarly to Fig. 3.14, the overall displacements are mostly positive; the only exception occurring at the middle of the longer edge. The entire slab experiences a uniform heat, due to the mid-surface temperature of 370°C. Without the effects of mechanical strains, the longer and shorter edges of the slab should have extended to 15.75mm and 10.5mm respectively.

**Fig. 3.22: Elevated-temperature rectangular slab in-plane displacement (long span)**

**Fig. 3.23: Elevated-temperature rectangular slab in-plane displacement (short span)**
These values were not attained because of the larger vertical displacement, which effectively pulled-in the edges and the corners.

Membrane tractions across the longer and shorter spans are shown in Figs. 3.24 and 3.25 respectively. The Rayleigh-Ritz model predicts compressive tractions in the central regions of the slab, across the longitudinal axes. This occurs in rectangular slabs because the longer edges are pulled inwards to a greater degree than the shorter edges.

![Graph showing membrane tractions](image)

**Fig. 3.24: Elevated temperature rectangular slab membrane tractions (long span)**

This inward movement therefore induces compression in the direction perpendicular to the longitudinal axis as the two edges approach each other. This behaviour was also observed by Foster (2006) on comparison of test deflections with *Vulcan* predictions.
Fig. 3.25: Elevated-temperature rectangular slab membrane tractions (short span)

Bottom and top layer stress distributions along the long span are shown in Figs. 3.26 and 3.27. The corresponding stresses in the bottom and top layers along the shorter span are shown in Figs. 3.28 and 3.29 respectively. Again, it is observed that the analytical model fails to predict stresses accurately towards the edges of the simply-supported slabs. From Figs. 3.26 to 3.29, it can be seen that the model provides a better prediction of the stresses in the longitudinal direction, than in the transverse direction.

Fig. 3.26: Elevated-temperature rectangular slab bottom layer stress (long span)
Chapter 3: Thermal Gradients

Fig. 3.27: Elevated-temperature rectangular slab top layer stress (long span)

Fig. 3.28: Elevated-temperature rectangular slab bottom layer stress (short span)
The Rayleigh-Ritz model gives reasonably accurate predictions of deflections and membrane tractions at ambient and elevated temperatures. The same cannot be said for stresses. The results show that the model provides good predictions of the stress distributions at the centre of the slab, but fails to replicate this at locations near the edges of the slab, especially at corners. The reason for the defects of the model is discussed in Section 3.6.

### 3.6 Discussion

In the generation of the analytical model, functional expressions were defined for the total mid-surface strains in the x- and y- directions and for the out-of-plane displacement. The full Fourier series expansion was used for the total strains, to allow in-plane displacement of the slab corners. However, the Fourier series expansion for the out-of-plane displacement was limited to the use of odd-numbered terms in cosine functions only.

The coupling of the highly nonlinear strain and potential energy equations, in addition to the use of the full and odd-numbered terms of the Fourier series expansions for the respective in-plane strain and out-of-plane displacement expressions make it difficult to simplify the in-plane shear strain, \( \gamma_{xy} \), for adaptation to a numerical process rather than the symbolic manipulation it required. Obtaining the amplitudes of the various shape-
functions through the solution of the simultaneous equations (performed with MAPLE 9.5) was therefore a very time-consuming process.

As is observed with finite element analyses, the introduction of higher degrees of freedom produces better results; the introduction of higher degrees of freedom in this process requires the inclusion of more terms of the Fourier series. The difficulty in performing the calculations, however, only allowed a limited number of expressions.

As a demonstration of the improvements that could be obtained with more expressions, the deflections, membrane tractions and stresses of a horizontally-unrestrained 5m x 5m square slab heated uniformly from 20°C to 720°C are monitored by using 16, 25 and 36 mid-surface total strain functions in each direction, while maintaining the number of out-of-plane functions at 4. Therefore, the total numbers of variables in these analyses were 36, 54 and 76 respectively. The results of the analyses are plotted in Figs. 3.30 to 3.32.

The results are coded in the usual colours, with dark blue representing the results along the slab centreline (y = 0), green representing the results along the slab edge (y = h/2) and red for the results along the line, equidistant from these two lines. The results for the analysis with 16 in-plane strain functions in one direction are shown as dotted lines, those for 25 in-plane strains are shown as dashed lines, and the more refined analysis with 36 in-plane functions (in one direction) shown as solid lines.

The uniform thermal expansion of the slab is expected to displace the edge of the slab by 17.5mm (Fig. 3.30). It is observed that all the analyses seem to converge on this number, indicating the good performance of the Rayleigh-Ritz method for displacements.
The variation of membrane tractions and stresses with the different numbers of functions is shown in Figs. 3.31 and 3.32. As the slab is unrestrained, the uniform application of heat should not induce any stresses in the slab. However, it can be seen from the two figures that significant stresses (and tractions) appear to be present in the slab with the smallest total number of expressions.

The results are significantly improved when the total number of variables increases from 36 to 54. However, even with a total variable count of 76, there are still some stresses in the slab, illustrating the point that a very large number of expressions is required to generate sufficiently accurate results with the Rayleigh-Ritz method.
Chapter 3: Thermal Gradients

Fig. 3.31: Membrane tractions for a uniformly heated unrestrained slab

Fig. 3.32: Stress distribution for a uniformly heated unrestrained slab
3.7 Summary

Differences in the development of tensile membrane action in horizontally-unrestrained simply supported concrete slabs have been recorded by previous researchers and in the previous chapter. The significant observation of the large deflections obtained with thermal gradients acting alone led to the development of an analytical method of quantifying the contribution of these to the development of tensile membrane action at elevated temperatures.

The method has shown that differential thermal expansion through the depth of a simply-supported slab can induce a considerable amount of tensile membrane action. It has also shown that it is not necessary to design edge beams and their connections to provide lateral restraint, as the compressive ring develops irrespective of the horizontal restraint along the boundary. The analytical model is found to be deficient where stresses along the boundaries of a slab are concerned. It has been found that an increase in the numbers of functions used in the analysis helps to improve the model's prediction. However, the numerical process has been difficult to implement due to the nature of the functions, and the coupled and highly nonlinear nature of the equations involved.

The method can be simplified further if a single shape function can be used for the out-of-plane deflection. This simplification will allow the generation of a numerical process that permits the incorporation of very large numbers of shape-functions, thereby increasing the accuracy of the predictions.
4 Slab stress patterns at elevated temperatures

Chapter 3 examined the development of tensile membrane action in vertically supported slabs exposed to thermal actions, acting alone. The analytical study, conducted with the Rayleigh-Ritz method, in comparison to numerical analyses by Vulcan and ABAQUS, confirmed the occurrence of the mechanism when a sufficient thermal gradient is induced through the depth of the slab. The analytical approach was originally intended to be used in further studies on stress patterns under different thermal exposures and boundary conditions. However, due to the inability of the Rayleigh-Ritz method to effectively represent stresses in slabs, especially towards the edges, due to the huge computational effort required, Vulcan is used for this extension in this chapter. The chapter therefore explores stress distributions in simply-supported slabs exposed to thermal gradients, and investigates the effects of varying through-depth thermal distributions, as well as axial restraints, on these stresses.

4.1 Stress distributions in horizontally unrestrained slabs

Chapter 3 showed that thermal gradients could induce tensile membrane action in thin slabs. In the analytical study, however, only a 7°C/mm thermal gradient was investigated. A non-degrading linearly-elastic concrete material model was used, so that the effects of the thermal gradient alone could be monitored, and also because the Rayleigh-Ritz approach considered the slab as a single homogeneous layer. Advanced software, such as Vulcan or ABAQUS, uses layered slabs. In this case different material properties and stress-strain characteristics can be used for each layer. The investigation in Chapter 3 is therefore extended to observe the effects of material degradation on the displacements, membrane tractions and stresses in the slab; the study is conducted with a linear thermal distribution through the depth of the slab. Subsequently, the effects of nonlinear distributions are investigated.

To isolate the effects of aspect ratio on the results, the investigation in this chapter is conducted on square slabs of 5m side length. As in the previous chapter, the slab is 100mm thick, with Young’s modulus of 18000MPa and Poisson’s ratio of 0.25, with a constant coefficient of thermal expansion of $10 \times 10^{-6}/°C$. One quarter of the slab is modelled in the finite element analysis, due to the symmetry of the problem. The quarter-slab is divided into 100 equal elements with 10 layers through each. The temperature of each layer is different, but constant within the layer, providing unique
elastic stress-strain characteristics. Eurocode 4 Part 1.2 reduction factors for the elastic modulus of concrete were used (CEN, 2005b). The thermal gradients were induced by keeping the slab top surface at 20°C while the bottom surface temperature was increased steadily from 20°C to 720°C at 5°C intervals. The results were recorded at thermal gradients between 1°C/mm and 7°C/mm.

Fig. 4.1 shows the vertical displacements of the horizontally-unrestrained quarter slab section along its centre line. In Fig. 4.1, and subsequent graphs in this chapter, unless otherwise stated, the results obtained at thermal gradients of 1, 2, 3, 4, 5, 6 and 7°C/mm are shown in dark blue, pink, green, red, violet, brown and teal respectively. Legends are also provided to highlight this, and to show from which section of the slab the results are taken; these locations are marked in black on the legend. The locations conform to the guide provided in Fig. 3.4. From Fig. 4.1 it is observed that the deflections of the slab at a thermal gradient of 7°C/mm are lower than those of the corresponding slab with the non-degrading material (Fig. 3.13). An examination of the membrane tractions and the stresses in the slab helps to explain this phenomenon.

![Fig. 4.1: Vertical displacements along centreline due to thermal gradients](image)

Fig. 4.2 shows the membrane tractions across a span of the slab. These tractions are highest across the centreline, and reduce to zero across the slab’s edges. The enclosure of the central tensile tractions by the ring of compression clearly indicates the presence of tensile membrane action.
Chapter 4: Slab stress patterns at elevated temperatures

Fig. 4.2: Membrane tractions along centreline due to thermal gradients

The magnitudes of the tractions are observed to be about one-fifth of the values recorded for the same slab with the non-degrading linearly-elastic material (compare values for the central membrane traction plot at 7°C/mm in Fig. 3.15).

An investigation of the stress distribution in the bottom and topmost layers presents an explanation of this behaviour. Figs. 4.3 and 4.4 show the respective bottom and top layer stress distributions in the x-direction along the centreline of the slab.

Fig. 4.3: Bottom layer stress distribution along centreline due to thermal gradients
Fig. 4.4: Top layer stress distribution along centreline due to thermal gradients

In general, the higher temperatures in the lower layers of the slab cause rapid thermal expansion against the cooler layers above. This behaviour induces compressive stresses in the lower layers, which gradually change through the depth of the slab, to tensile stresses in the upper layers. This stress distribution is predominant in the centre of the slab, where the cumulative effect results in tensile tractions (Fig. 4.2). As one moves from the centre towards the edges, compressive stresses increase as the peripheral compressive ring is approached. The stresses then drop to zero at the horizontally-unrestrained boundary.

From Fig. 4.3, it can be seen that comparatively lower compressive stresses are recorded in the bottom layer. The results also show that increasing the thermal gradient does not always increase the induced compressive stress, in contrast to what is observed in the topmost layer (Fig. 4.4). The effect is more pronounced when the stresses along \( y = b/4 \) and the boundaries are examined in Figs. 4.5 – 4.8. Increases in the magnitudes of the compressive stresses (in the bottom layers) are observed when the thermal gradients increase from 1°C/mm to 4°C/mm. Thereafter, further increases in thermal gradient cause a reduction of stresses, especially towards the centre of the slab. This is because of the reduction of the elastic modulus of the lower layers.

The linear stress-strain characteristics and thermal expansion model employed in the study are based on lightweight concrete properties, as given in Eurocode 4 Part 1.2 (CEN, 2005b). At a 4°C/mm gradient, the bottom layer is at a temperature of 400°C.
The tangent modulus, which depends on the concrete strength \( f_{cu,\theta} \), the instantaneous strain \( \varepsilon_{c,\theta} \) and the strain at maximum stress \( \varepsilon_{cu,\theta} \), simplifies to the elastic modulus at zero instantaneous strain, as shown in Equation 4.1 below:

\[
E = \frac{3f_{cu,\theta}}{2\varepsilon_{cu,\theta}}
\]

(4.1)

Therefore, at 20°C the Young’s modulus is 18000N/mm\(^2\). At 400°C, however, lightweight concrete loses 12% of its strength and there is a 300% increase in the strain at ultimate stress. The Young’s modulus therefore reduces to 3960N/mm\(^2\).
Fig. 4.7: Bottom layer stress distribution along edge due to thermal gradients

Fig. 4.8: Top layer stress distribution along edge due to thermal gradients

Any increase in strain due to thermal expansion therefore generates very little compressive stress in the lower layers. Since the lower layers induce the tensile stresses in the top layers, a reduction in the magnitudes of compressive stresses causes a general reduction in tensile stresses at the top, thereby affecting the membrane tractions, which vary nonlinearly with vertical displacement. This explains the marked reduction in vertical deflections in comparison with those obtained by the non-degrading concrete model.
The study has therefore shown that material degradations reduce stresses in degraded parts of the slab, which lower membrane tractions and the resulting deflections.

### 4.2 Effects of nonlinear thermal gradients

Due to the lower conductivity of concrete and the fact that slabs are mostly exposed to fires on one side, their typical thermal distributions in fire are nonlinear. This section therefore explores the effects of a bilinear thermal exposure in the slab cross-section, and compares the behaviour of slabs exposed to the two thermal distributions. The surface temperatures in the two models are kept the same. The investigation initially examines the effects of the gradients only, with a non-degrading material model. The influence of material degradation is subsequently considered. Fig. 4.9 shows the two through-depth temperature distributions used in the study. The temperatures of the top surface, mid-surface and bottom surface of the slabs with the two temperature distributions, at various thermal gradients are given in Table 4.1.

![Fig. 4.9: Linear and bilinear thermal distributions](image)

<table>
<thead>
<tr>
<th>Thermal distribution</th>
<th>Thermal Gradient (°C/mm)</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<td></td>
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<td>320</td>
<td>420</td>
<td>520</td>
<td>620</td>
<td>720</td>
</tr>
</tbody>
</table>

**Table 4.1: Temperature distributions**
4.2.1.1 Non-degrading material

Vertical displacements of the centrelines in the two models, at mean thermal gradients of 1°C/mm, 4°C/mm and 7°C/mm, are shown in Fig. 4.10. The deflections are observed to be the same. A plot of the in-plane displacements along the centreline (Fig. 4.11) however shows that the horizontal displacements of the bilinear model are less than those of the linear model. In *Vulcan*, nodes are located at the mid-surface of the slab. The lower temperatures at the mid-surfaces of the bilinear models imply lower thermal expansion in comparison with the linear models. This therefore affects the in-plane displacement.
Chapter 4: Slab stress patterns at elevated temperatures

Membrane tractions across the centreline, recorded at the three thermal gradients of the two models, are presented in Fig. 4.12. It is seen that the results of the two models are the same once more. Examinations of the bottom- (Fig. 4.13) and top-layer (Fig. 4.14) stress distributions, however, show that the reduced thermal expansion of the mid-surface in the bilinear model induces higher compressive stresses in the lower layers. The relatively cold mid-surface also prevents excessive stretching of the top layers, thereby reducing the tensile stresses induced in them.

Fig. 4.12: Membrane tractions of thermal models (non-degrading material)

Fig. 4.13: Bottom layer stress distribution of thermal models (non-degrading material)
Chapter 4: Slab stress patterns at elevated temperatures

It is observed that the differences between the top and bottom stress distributions in the two independent models are equal. The cumulative effects of the stresses in all 10 layers therefore yield the equal membrane tractions and vertical displacements, as observed in Figs. 4.10 and 4.12.

### 4.2.1.2 Degrading material

Fig. 4.15 shows the membrane tractions of the two models across the centreline of the slab. Again, it can be seen that the two models produce identical results. The bottom
and top layer stresses along the slab centrelines, shown in Figs. 4.16 and 4.17 reveal higher compressive stresses in the bilinear model, primarily due to the lower thermal expansion of the mid-surface. This induces compressive stresses generally throughout the depth of the slab. Unlike the model with the non-degrading concrete material, the algebraic difference between the top and bottom layers is higher in the linear temperature distribution model than the bilinear model. This therefore induces larger deflections in the linear model, as shown in Fig. 4.18. Fig. 4.19 shows the in-plane displacements of the two models at the various thermal gradients.

**Fig. 4.16: Bottom layer stress distribution of thermal models (degrading material)**

**Fig. 4.17: Top layer stress distribution of thermal models (degrading material)**
The results show that the bilinear thermal distribution generally produces lower deflections. However, larger compressive stresses are induced through the thickness of the slab as a result of the smaller elongation of the mid-plane in comparison to the rapid expansion of the lower layers. The smaller elongation of the mid-plane therefore acts like an artificial in-plane restraint to thermal expansion.

Fig. 4.18: Vertical displacements of thermal models (degrading material)

Fig. 4.19: In-plane displacements of thermal models (degrading material)
4.3 Boundary restraints and their effects on slab behaviour in fire

Composite slabs are exposed to two main thermal actions; these are thermal expansion and thermal gradients. Fire exposure at the slab soffit introduces differential thermal expansion through the depth of the slab, which causes thermal bowing towards the fire and elongations of the slab edges against any boundary restraints that may be present. As concrete members heat up slowly, they are sensitive to axial restraints. Depending on the magnitudes of these restraints and their lines of action relative to the centroid of the slab, they can be either advantageous or detrimental to slab capacities at elevated temperatures.

The rapid thermal expansion at the bottom of a slab against supports that resist lateral movement (as shown in Fig. 4.20) introduces compressive forces which produce a moment $T_e$ which enhances the slab resistance against bending. In Fig. 4.20, $T$ is the axial restraint force and $e$ is the distance between the centroid of the slab and the location of the resultant axial thrust. It is obvious from the figure that $T_e$ reduces vertical displacements as long as the thrust is below its centroid, and increases vertical displacements otherwise. With increasing vertical deflections, however, the induced compressive forces are lost, and fire resistance can only be maintained by strong vertical supports and sufficient anchorage of reinforcement into the surrounding structure (Buchanan, 2001).

![Fig. 4.20: Effect of axial restraint force (Buchanan, 2001)](image)

Research on the effects of axial restraint on reinforced concrete slab behaviour in fire has been conducted by Lim (2003). This involved studies on the effects of the position of the line of thrust of the compressive force on one-way spanning slabs. By examining various axial thrust locations, using the specialist fire engineering finite element software SAFIR, the effect of the location of the compressive thrust was confirmed.
A subsequent study on the effects of different axial restraint stiffnesses, also on one-way slabs, revealed that the rate of development of compressive axial force in the slab increased with increasing restraint stiffness. Also, low axial stiffness allowed larger vertical deflections of the slab, requiring loads to be borne by catenary action. Higher restraint stiffnesses significantly reduced vertical displacements, causing hogging deflected shapes in some cases. A further investigation showed that rotational restraints at the ends of the slab significantly increased its resistance, and made the effects of different axial restraint stiffnesses negligible (Lim 2003; Lim et al., 2004b).

Composite slabs are generally thinner than conventional reinforced concrete slabs. The increasing use of performance-based approaches at elevated temperatures implies that these slabs are designed to span in two directions, so that tensile membrane action can enhance their resistance. In general this mechanism does not require axial or rotational restraints, but these can be beneficial at large deflections.

In the initial stages of a fire, restraint to the mean thermal expansion of a slab will be counteracted by the induced thermal gradient until deflections become so high that all compressive axial force is released. Also, with rotational restraint along the edges of the slab, thermal deflections due to the thermal gradients will be resisted. These opposing influences can either aid or resist deflections of the structure, depending on the degree of restraint to the fire-exposed slab. The subsequent sections in this chapter therefore examine how these boundary restraints affect the development of tensile membrane action in-terms of vertical displacements, membrane tractions and stresses.

As applied loads increase vertical displacements, the investigation isolates the effects of vertical loads by considering the effects of thermal actions only, as in previous sections. Thermal gradients between 1 °C/mm and 7 °C/mm are again examined. The behaviour of slabs exposed to a linear temperature distribution is examined. The investigation is carried out with degrading linearly-elastic material behaviour.

### 4.3.1 Properties of the restrained slab

The series of studies were conducted with *Vulcan*, the University of Sheffield’s specialist structural fire engineering software (Huang et al., 2003a; 2003b). The same 100mm thick flat slab, of dimensions 5m x 5m, was used for the analyses. The vertically supported edges of the slab were connected to axial in-plane springs. The springs allowed the definition of different axial stiffnesses. No transverse loads were
placed on the slab except for a small load of 0.0001kN/m$^2$, which prevented upwards vertical bifurcation and offered numerical stability to the nonlinear solution. Due to the inherent symmetry of the problem, one-quarter of the slab was modelled. Fig. 4.21 shows the quarter-section with the 100 *Vulcan* slab elements and the in-plane restraints. The origin was set at the centre of the slab, as shown in the figure.

![Vulcan finite element quarter section model](image)

The restraint stiffnesses along the edges of the slab were generated as multiples of the axial stiffness of the slab in one direction. This was calculated as

\[
\frac{EA}{L} = \frac{18000 N/mm^2 \times 5000 mm \times 100 mm}{5000 mm} = 1800000 N/mm
\]  

(4.2)

On each side, of length 5m, there were 41 springs; therefore each axial spring stiffness was 43902.44N/mm. The axial springs were attached to the slab elements at mid-depth. Five different axial spring stiffnesses were tested. These were 0.25, 0.5, 1.0, 2.0 and 4.0 times the axial stiffness of the slab. A horizontally unrestrained slab, and one with all axial displacements fixed, were used as the limiting cases.

### 4.3.2 Effects of axial restraint

Fig. 4.22 shows the variation of the central vertical displacements of the axially restrained slabs with linear thermal gradients. It is observed that deflections at various gradients increase with increasing axial restraint.
Chapter 4: Slab stress patterns at elevated temperatures

Fig. 4.22: Vertical displacements vs. thermal gradients at various axial restraints

Fig. 4.23 shows the bottom layer stress distribution along the centreline of the slab that is axially restrained by 0.25 times the slab’s axial stiffness. The stress distributions obtained with the 7 thermal gradients are shown in Fig. 4.23.

Fig. 4.23: Bottom layer stress distribution with 0.25x slab axial stiffness

Increasing thermal gradients are expected to produce corresponding increases in the magnitudes of the compressive stresses in the bottom layer. An increase in compressive stress is seen from 1 °C/mm to 2 °C/mm; higher gradients result in decreases in compression in the bottom layer. An examination of the top layer
stresses reveals however that increasing thermal gradients increase the tensile stress (Fig. 4.24). The forces in the axial springs along the centreline for the various axially restrained slabs are shown in Fig. 4.25.

![Stress Distribution](image1)

**Fig. 4.24: Top layer stress distribution with 0.25x slab axial stiffness**

![Spring Force](image2)

**Fig. 4.25: Axial spring forces for different thermal gradients for various axial restraint stiffnesses**

It is observed from Fig. 4.25 that compressive forces develop in the spring from zero gradients up to about a gradient of 4 °C/mm, before the compressive force is released. The thermal gradients in the slab tend to induce vertical displacements in the slab, while...
the axial restraints tend to restrain the slabs from doing so. The complex mechanism is explained after the examination of the slab with 1.0x axial restraint. The membrane tractions of the 0.25x axially restrained slab are shown in Fig. 4.26.

![Fig. 4.26: Membrane tractions for different thermal gradients with 0.25x slab axial stiffness](image)

Figs. 4.27 and 4.28 show the respective bottom and top layer stress distributions of the axially supported slab restrained by 1.0 times the slab’s axial stiffness. It is observed that, in the bottom layer, there are reductions in compressive stresses from thermal gradient 1 °C/mm to 2 °C/mm, with some parts of the bottom layer going into tension at 3 °C/mm. However, beyond 4 °C/mm the compressive stresses, especially in the centre of the slab, start increasing again. Fig. 4.28 shows a similar trend for the top layer stresses.

Fig. 4.25 shows the spring force in the slab restrained with the same restraint, gaining peak compression just beyond a thermal gradient of 2 °C/mm. The compressive force reduces for higher thermal gradients. It can be speculated that:

1. The initial thermal gradients, regardless of how small they are, induce compressive stresses in the bottom layers and tensile stresses in the top layers.
2. Further increases in thermal gradients are expected to increase the magnitudes of stresses in these layers. However, the uniform expansion which accompanies the thermal gradient (expansion of the mid-surface) is restrained by the in-plane stiffnesses at the boundaries.
3. This causes a reduction in stresses in certain areas, as indefinite thermal expansion cannot be maintained, until a point is reached at which the
combination of the induced thermal gradient and the mean thermal expansion cause thermal buckling, which in turn releases the compressive axial restraint

4. Subsequently, the thermal gradient dominates the behaviour of the slab once again, re-introducing compressive stresses into the lower layers, with corresponding tensile stresses in the layers above.

Fig. 4.27: Bottom layer stress for different thermal gradients with 1.0x slab axial stiffness

Fig. 4.28: Top layer stress for different thermal gradients with 1.0x slab axial stiffness
Fig. 4.29: Membrane tractions for different thermal gradients with 1.0x slab axial stiffness

Fig. 4.29 shows the membrane tractions of the slab with restraint stiffness 1.0x the axial slab stiffness. It is observed from the plot and Fig. 4.26 that central tensile tractions only begin to form after the axial compressive forces have been released (Fig. 4.25). The same phenomenon is used to explain the stress distributions in the bottom (Fig. 4.30) and top (Fig. 4.31) layers of the slab restrained by 4 times the axial slab stiffness. It is observed that the compressive stresses become dominant again in the bottom layer beyond a thermal gradient of 3 °C/mm.

Fig. 4.30: Bottom layer stress for different thermal gradients with 4.0x slab axial stiffness
Fig. 4.31: Top layer stress for different thermal gradients with 4.0x slab axial stiffness

Membrane tractions of this restrained slab are shown in Fig. 4.32. It is observed that the maximum tensile tractions are approximately the same regardless of the axial restraint. However, the maximum compressive tractions seem to increase with increasing axial restraint.

Fig. 4.32: Membrane traction for different thermal gradients with 4.0x slab axial stiffness
4.4 Summary

This chapter has extended the study on the effects of thermal gradients on tensile membrane action, using thermal gradients from 1°C/mm to 7°C/mm. The study investigated the influence of material degradation on stresses, membrane tractions and displacements. Horizontally unrestrained slabs were examined, as well as those horizontally restrained to various degrees. Linear and bilinear gradients were also investigated.

In general, thermal gradients induce compressive and tensile stresses in the bottom and top layers of a slab respectively. These stresses increase with increasing gradient. However, in the presence of axial restraint the gradients become dormant until a point at which the mean thermal expansions generate enough vertical deflections to release the induced compressive stress. It is only after this that thermal gradients can positively influence the behaviour of the slabs by generating sufficient tensile membrane action.
5 Slab Panel vertical support

The advancement in structural fire engineering towards more cost-effective solutions has necessitated the increasing use of performance-based approaches to the design of multi-storey composite buildings. These methods consider the real behaviour of structures and, as such, are more economically viable alternatives if fire protection is optimised. Optimising structures for tensile membrane action requires the use of slab panels. These are vertically-supported floor systems allowing biaxial bending at elevated temperatures. Vertical support is achieved, in practice, by protecting a panel’s perimeter beams to achieve temperatures of no more than 620°C at the required fire resistance time. With exposure to fire, this support can be lost, losing the panel’s tensile membrane capacity as a result; and pre-empting failure of the floor system. The resulting mechanism, if not adequately designed, can lead to a catastrophic failure of the structure.

This chapter presents a finite element investigation into the provision of adequate vertical support along slab panel boundaries. The study examines various degrees of protection relative to the development of the membrane mechanism. It examines the development and failure of the membrane mechanism, considering various degrees of edge-beam protection, and makes comparisons with predictions of the membrane action design method and various design acceptance criteria. The findings of this chapter have been presented at two conferences (Abu et al., 2007; 2008a) and have been published in the Steel and Composite Structures Journal (Abu et al., 2008b).

5.1 Design of Composite Slabs in Fire

5.1.1 Traditional Fire Engineering Design
The trend in fire engineering has been to apply fire protection prescriptively to all exposed steelwork in a building after room temperature design, to achieve a fire resistance rating specified on the basis of the height and use of the building (Approved Document B, Volume 2, 2007). This design methodology stems from the assumption that individual structural elements behave independently in fire, ignoring interactions that may be present between various parts of the structure. Research, and observations of structural behaviour under fire conditions over the past 20 years, have shown that load redistribution and large deflections of parts of the structure at the Fire Limit State are essential to the survival of the entire structure (Usmani et al., 2001).
In particular, it has been observed that optimising composite floors for tensile membrane action yields considerable savings in protection costs, and structural stability is obtained by taking advantage of real building behaviour during fires. The conditions necessary for the effective use of this mechanism are two-way bending and vertical support along the edges of the slab. Due to its self-equilibrating nature, horizontal edge restraint is not required for the mobilisation of tensile membrane action, as observed in Chapter 3.

5.1.2 Performance-based approach
To optimise these composite floors to take advantage of this higher load capacity in structural fire engineering design, a composite floor is divided into several fire-resisting rectangular zones of low aspect ratio, called slab panels; each comprising a set of adjacent unprotected composite beams in the interior of the panel, with edges that primarily resist vertical deflection. This vertical support is usually provided by protected beams along all four edges, and these panels are generally set out to lie between column gridlines as shown in Fig. 5.1. In fire the unprotected beams lose strength and stiffness rapidly, and their loads are borne by the composite slab, which increases in capacity as its deflection increases.

![Fig. 5.1: Typical composite slab panels](image)

5.1.3 Simple and Sophisticated Performance-based models
Tensile membrane action, and whole-structure behaviour at high temperatures, can be modelled in a three-dimensional framework with sophisticated finite element software such as *Vulcan* (Huang *et al.*, 2002b; 2003a; 2003b; 2004a), TNO DIANA and
ABAQUS. Although finite element simulations provide useful information on complete load-deformation and stress development at elevated temperatures, they can be very costly processes. As such, simpler performance-based methods, like the Bailey-BRE Membrane Action Method (which can easily be set up as a spreadsheet), are often preferred for routine design. However, the simplifications applied in some of these approaches can lead to unrealistic or over-conservative designs as discovered by Huang et al. (2002a) when a comparative study of the Bailey-BRE method and Vulcan was first undertaken.

Firstly, a 9m x 9m, 130mm deep, ribbed reinforced concrete slab was analysed by Huang et al. (2002a), using Vulcan and the Bailey-BRE method. For accurate like-against-like comparisons, a linear temperature profile was used through the depth of the concrete slab. In the Vulcan analyses, two horizontal support conditions were investigated; simple supports (allowing horizontal pull-in) and hinge supports (preventing horizontal pull-in). The results showed that although the Bailey-BRE method was based on simply-supported edges, it presented capacities similar to the hinge-supported Vulcan slab, especially towards failure. The second comparison examined the effects of using higher reinforcement ratios on slab panels of aspect ratio 1.0 and 2.0. Disproportionately large capacities were observed in the Bailey-BRE slab panel with an aspect ratio of 1.0 when reinforcement mesh size was increased from A142 to A393. A comparison with the Vulcan slab panel showed moderate increases in capacity even with cold perimeter beams. It was however, observed that the comparison of models with the A393 mesh was comparable between the two models for the panel with the 2.0 aspect ratio.

Chapters 5 and 6, therefore, examine the credibility of the Bailey-BRE method through the use of finite element studies, with the aim of establishing how a slab panel’s vertical support and reinforcement ratio affects its capacity in fire. Chapter 5 concentrates primarily on the issue of vertical support while Chapter 6 addresses the effects of reinforcement ratios. These comparative studies aim to assess the efficiency of the Bailey-BRE method as a tool for preliminary investigation of slab behaviour in fire.

5.2 Bailey-BRE membrane action method

5.2.1 Description

The Bailey-BRE method (Bailey, 2000; 2001; 2003; 2004) proceeds by dividing a composite floor into an array of horizontally-unrestrained, vertically supported slab
panels. Each of these panels is composed internally of simply-supported unprotected beams. With increasing exposure to elevated temperatures, the formation of plastic hinges in the unprotected beams re-distributes the loads to the two-way bending slab, undergoing large vertical deflections. Based on rigid-plastic theory with large change of geometry, the additional slab capacity provided by tensile membrane action is calculated as an enhancement of the small-deflection yield-line capacity. Failure is determined by the formation of a full-depth tension crack across the shorter span of the slab. The method conservatively ignores any contribution of the tensile strength of concrete to the capacity of the slab, and does not provide any information on the state of the protected boundary beams, apart from the assumption that they remain vertically undeflected throughout a fire.

The Bailey-BRE method, initially developed for isotropic reinforcement (Bailey, 2000, 2001), has since been extended to include orthotropic reinforcement (Bailey, 2003). Recently the changes of in-plane stress distributions, and crushing failure due to the ring of compressive membrane stress, have been added (Bailey and Toh, 2007).

The procedure, developed from room-temperature conditions, assumes that the tensile membrane action mechanism at ambient temperature is maintained at elevated temperatures (Bailey, 2000). Research has, however, shown that the development of tensile membrane action at elevated temperatures differs from the ambient-temperature development (Cameron and Usmani, 2005a; 2005b; Foster, 2006; Abu et al., 2006).

### 5.2.2 SCI P-288 and TSLAB

The Steel Construction Institute (SCI) prepared a design guide (P-288), which lists tables of minimum reinforcement mesh sizes required to satisfy an allowable deflection limit criterion ($v$) at a defined fire resistance time (Newman et al., 2006). This limit is based on the mechanical strain allowed in the reinforcement at fracture and thermal bowing in the slab, as observed from Equation 1.3, repeated here as Equation 5.1.

$$v = \frac{\alpha(T_2 - T_1)}{19.2h} + \sqrt{\frac{0.5f_y}{E_{r=20^\circ C}}} \times \frac{3L^2}{8}$$  \hspace{1cm} (5.1)
5.2.3 Panel Vertical Support

In practice, slab panel vertical support is achieved by protecting the beams around the perimeter of each panel. The assumption of continuous vertical restraint at all times during the fire is therefore unrealistic. At any point during the fire, a combination of imposed loads and loss of strength and stiffness of the perimeter beams will induce vertical displacements, allowing the eventual formation of a single-curvature slab-bending mechanism. The slab panel will then hang from its connections, leading to a catastrophic failure of the structure if these connections are not adequately designed against such forces.

The potential for this type of failure has led to the series of finite element studies reported here, into the adequacy of vertical support along slab panel boundaries. The study is conducted with Vulcan and the Bailey-BRE method to examine some of its inherent assumptions, and the implications of the lack of adequate vertical support along the slab panel boundary.

5.3 Finite Element Study

5.3.1 Slab Panel Properties

A slab panel of dimensions 7.5m x 9.0m with a 60-minute fire resistance period was chosen, using normal-weight concrete and the trapezoidal slab profile shown in Fig. 5.2. The panel had secondary beams spanning in the shorter direction, with its primary beams spanning in the longer direction (Fig. 5.3). The secondary beams were at 3m spacing. According to SCI P-288 (Newman et al., 2006) the required minimum S500 reinforcement mesh size for the slab panel, with the design loading given in Table 5.1, was A193 (193mm²/m in each direction). This allowed for additional distributed line-loads of 20kN on the protected secondary beams and a maximum intermediate beam design factor of 1.00.
Fig. 5.2: Concrete slab cross-section, showing the trapezoidal decking profile

(a): Top face  
(b): Bottom face

Fig. 5.3: The 7.5m x 9.0m Slab Panel

<table>
<thead>
<tr>
<th>Permanent Loading</th>
<th>kN/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slab self-weight</td>
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</tr>
<tr>
<td>Beam self-weight</td>
<td>0.20</td>
</tr>
<tr>
<td>Reinforcement</td>
<td>0.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Imposed Loading</th>
<th></th>
</tr>
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<tbody>
<tr>
<td>Variable load</td>
<td>3.5</td>
</tr>
<tr>
<td>Ceilings/ Services</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 5.1: Slab panel design loading
5.3.1.1 Design
Ambient- and elevated-temperature design of the floor beams was carried out using BS 5950 Part 3 (BSI, 1990) and BS 5950 Part 8 (BSI, 2003), and assuming full composite action. This resulted in the choice of 356x127x33 UB and 533x210x82 UB as the secondary and primary beams respectively (Fig. 5.3). The actual design safety factor of the intermediate beams was calculated as 0.74 (factored loading of 11.66kN/m² and 6.08kN/m² at ambient and elevated temperatures, respectively).

Neither the Bailey-BRE Method nor TSLAB analyses the protected beams once the additional imposed loads at the fire limit state are accounted for in their elevated-temperature single-element design. The finite element analyses, however, include the effects of thermal exposure of the perimeter beams. Therefore, with guidance from the ASFP Fire protection guidelines (ASFP, 2002) and steel temperature calculations from Eurocode 3 part 1.2 (CEN, 2005a), a perimeter protection scheme was adopted with lightweight fire-resisting gypsum boards (density = 800kg/m³; specific heat capacity = 1700J/kg/K; conductivity = 0.2W/mK) to ensure that protected beam temperatures were limited to a maximum of 550°C at 60 minutes. The finite element analyses were performed with Vulcan (Huang et al., 2002b; 2003a; 2003b; 2004).

Table 5.2 presents details of load ratios and limiting temperatures of the protected beams used in the finite element analyses.

<table>
<thead>
<tr>
<th>Protected Beam type</th>
<th>Beam Section</th>
<th>Load Ratio</th>
<th>Limiting Temperature</th>
<th>Temperature at 60mins</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secondary</td>
<td>356 x 127 x 33 UB</td>
<td>0.440</td>
<td>631°C</td>
<td>537°C</td>
</tr>
<tr>
<td>Primary</td>
<td>533 x 210 x 82 UB</td>
<td>0.396</td>
<td>647°C</td>
<td>542°C</td>
</tr>
</tbody>
</table>

Table 5.2: Protected beam design properties

5.3.1.2 Vulcan
Vulcan is a geometrically nonlinear finite element program which includes the effects of nonlinear material behaviour at elevated temperatures. In Vulcan reinforced concrete slabs are modelled with 9-noded nonlinear layered rectangular elements, which represent temperature distributions through the depth of the slab by assigning different, but uniform, temperatures to each layer of the slab element. Bending and membrane effects at large displacements are also represented. Reinforcement is modelled as layers with uniaxial properties, being smeared across the area of the slab element. The program uses an elevated-temperature model of the biaxial concrete failure
surface based on a modification of the Kupfer and Gerstle expressions (1973). As a result it can represent failure in tension or compression. Beams are modelled using 3-noded nonlinear beam elements.

5.3.2 Thermal Analyses

A series of thermal analyses was conducted to ascertain the temperature distributions through the depths of the two (BRE-TSLAB and Vulcan) slab panel models with time. TSLAB performs two analyses to determine the distribution of temperatures through the slab, and in particular to calculate the temperature of the reinforcement. These consider sections through the slab at its thickest and thinnest points – i.e. through a rib and through the concrete topping respectively. A weighted average of these temperatures is then used. However, the generic Bailey-BRE method uses an average-depth flat concrete slab for its structural calculations (Bailey, 2000).

Two Vulcan models were therefore analysed under exposure to the standard temperature-time curve; the first was a two-dimensional model in which a weighted average of the reinforcement temperatures through the ribs and thinner parts was calculated; the second considered an equivalent solid slab of thickness equal to the average depth of the composite slab (100mm). The one- and two-dimensional thermal analyses were performed with the program FPRCBC-T (Huang et al., 1996).

TSLAB unprotected beam bottom flange temperatures were used for both the Bailey-BRE and Vulcan analyses. For both methods, non-uniform temperature distributions were used through the beam depths. The beam top flange temperatures were set to be 20% lower than those of their webs or bottom flanges, which were at the same temperature.

Comparisons of slab temperature distributions in the Vulcan and TSLAB models, for a 90-minute exposure to the standard temperature-time curve, are shown in Figs. 5.4(b) and 5.5(b) with reference to Figs. 5.4(a) and 5.5(a). TSLAB results are shown as broken lines while the Vulcan temperature distributions are shown as continuous lines. The colours and numbers on the temperature-time curves on the right hand side (Figs. 5.4(b) and 5.5(b)) correspond to the reference points on the schematic diagrams on the left hand side (Figs 5.4(a) and 5.5(a)).
It is observed from Figs. 5.4(b) and 5.5(b) that there was good correlation between the thermal distributions in the *Vulcan* and TSLAB models for the second analysis, in which an average depth of concrete was used. Consistent with assumptions of the basic Bailey-BRE method, a depth of 100mm was used for the structural analyses of the *Vulcan* and Bailey-BRE models. Beam bottom flange temperature distributions for the *Vulcan* models are shown in Fig. 5.6, along with the standard fire curve in red.
5.3.3 Structural Analyses

TSLAB does not output the fire resistance time that a particular slab panel arrangement can achieve. Instead, the user specifies the required fire resistance and the software checks if the model complies with the specification. Since the generic Bailey-BRE method was used in this research, TSLAB was only used to generate the allowable maximum vertical displacements with time for comparison, and a spreadsheet was written to simulate the calculations embodied in TSLAB whilst outputting results in a form suitable for direct comparison with the other approaches. However, TSLAB has been found not to accurately interpret the Bailey-BRE calculations (Toh and Bailey, 2007), so the original equations are used in the spreadsheet.

The primary *Vulcan* analysis attempted to simulate the behaviour of the actual slab panel vertical support in fire conditions. This was achieved by vertically restraining the corners of the model and preventing rigid body motion of the slab (as shown in Fig. 5.7), but allowing vertical displacements of the protected beams on the panel perimeter. The entire slab panel was restrained against rotation about the vertical z-axis.

Fig. 5.6: Temperature-time relationships for beams in the *Vulcan* analyses
For comparison, other *Vulcan* analyses were performed with differing edge support conditions to determine their effects on tensile membrane action. Variations, of which the details are given in Table 5.3, included:

- Rigid vertical support along the perimeter of the slab panel,
- Rotational restraint along the perimeter of the panel,
- Using twice the amount of generic protection on the perimeter beams,
- The assumption of cold perimeter beams.

<table>
<thead>
<tr>
<th>Condition</th>
<th>$V_1$</th>
<th>$V_2$</th>
<th>$V_3$</th>
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</tbody>
</table>

Table 5.3: *Vulcan* Analyses and Parameters
The results of the *Vulcan* analyses were compared with:

1. The TSLAB limiting deflection curve
2. The Bailey-BRE allowable vertical deflection limit
3. The required vertical deflection (from the generic Bailey-BRE approach)
4. A limiting deflection of short span/20 (7500mm/20 = 375mm)

Fig. 5.8 compares the vertical deflection criteria used in the study. The TSLAB deflection limit at each time-step is obtained from Equation 5.1. To obtain the Bailey-BRE allowable vertical deflection limit, $T_2 - T_1$ is assumed to be equal to 770°C for fire exposure below 90 minutes. The two Bailey-BRE deflections are also shown. The required deflections of the original Bailey-BRE method (Bailey, 2000) are shown as broken lines, while those of the improved method (Bailey and Toh, 2007a) are shown as a solid black line.

![Fig. 5.8: Bailey-BRE vertical displacements and other deflection criteria](image)

It is observed that the two Bailey method displacements compare very closely and both satisfy the TSLAB deflection limit. The difference between the two methods is noticeable with very high reinforcement ratios, where failure of the compression ring becomes critical. The updated formulation of the Bailey-BRE method (Bailey and Toh, 2007a) and the TSLAB limiting deflection (Equation 5.1) were used for all the comparisons in this chapter.
5.3.4 Results and Discussions

The results of the various *Vulcan* analyses are presented in this section. It should be noted that, unless otherwise stated, the results presented here all show absolute maximum vertical displacements of the centre of the 7.5m x 9.0m slab panel system reinforced with an A193 mesh.

5.3.4.1 Vertical restraint

Fig. 5.9 shows plots from the first two *Vulcan* analyses and the Bailey-BRE displacement. The $V_1$ and $V_2$ models were identical, except that $V_2$ included vertical support to its perimeter beams. The advantage of this continuous vertical support is immediately apparent with the Bailey-BRE method and $V_2$ giving vertical displacements of 401mm and 359mm respectively at 60 minutes, as compared with 576mm in the $V_1$ model. Fig. 5.9 also confirms that the idealised behaviour on which the Bailey-BRE approach is based is very unconservative, as it shows no sign of failure. According to the span/20 and BRE failure limits, the *Vulcan* $V_1$ slab panel fails at 26 and 40 minutes, respectively. The *Vulcan* analysis however shows ‘runaway’ failure accelerating after 70 minutes.

![Fig. 5.9: Comparison of Vulcan $V_1$, $V_2$ and the BRE displacement](image)

Fig. 5.10 examines the failure mode of *Vulcan* model $V_1$. Displacements at the centre of the slab panel relative to the vertical deflections at the midpoints of the protected secondary and primary beams are shown. It can be seen that there is a lessening difference as time goes on between the displacements of the middle of the slab and the
midpoints of the protected secondary beams, with a steady increase in the displacement of the middle of the slab panel relative to the primary beams. This reduction in relative displacements shows the accelerated deflection of the protected secondary beams before failure. This phenomenon is further illustrated in Figs. 5.11 to 5.14.

In the illustrations, beams are shown as solid red lines running across the length and width of the slab panel. Tensile tractions are shown in red while compressive tractions are shown in blue. At ambient temperature, the slab acts as the compressive flanges of the composite beams, as shown in Fig. 5.11(b). In fire, the loss of strength of the unprotected intermediate beams, and thermal bowing of the slab, allow the progressive development of tensile membrane action, until such a time as degradation of material properties and loss of stiffness cause failure.

![Fig. 5.10: Failure analysis of the Vulcan V1 model](image)

From Figs. 5.11 to 5.14, it is observed that after 70 minutes the loss of strength of the protected secondary beams results in the formation of a single-curvature bending mechanism, which causes the failure of the slab panel system and the loss of tensile membrane action. The temperature of the protected secondary beams at failure is 600°C. It is observed from the membrane traction plots of Fig. 5.14(b) that after 70 minutes the single-curvature mechanism causes compressive tractions in the shorter direction of the slab, confirming the loss of tensile membrane action. The behaviour of
the protected secondary beams is seen as the formation of fully-developed plastic hinges, thereby allowing the slab panel to effectively fold.

Fig. 5.11: $V_1$, Slab panel results at 0mins

(a) Deflection at time, $t = 0.0$ mins
(b) Membrane tractions at time, $t = 0.0$ mins

Fig. 5.12: $V_1$, Slab panel results at 60mins

(a) Deflection at time, $t = 60.0$ mins
(b) Membrane tractions at time, $t = 60.0$ mins
5.3.4.2 Influence of the tensile strength of concrete

The idealised slab panel behaviour portrayed in the Bailey-BRE method assumes that the tensile strength of concrete is negligible and that the panel boundaries remain cold. Fig. 5.15 therefore shows a comparison between analytical results for the Bailey-BRE model and for Vulcan slab panels with perimeter beams which are kept at 20°C throughout the fire and are vertically supported along their edges. Model V₃ includes the effects of the tensile strength of concrete, and predicts a greater slab capacity than the Bailey-BRE model. However, its capacity measured against deflection limits is less
than that of the *Vulcan* $V_2$ model. This is because the restrained thermal expansion of the concrete slab against the consistently cold perimeter beams causes higher deflections; this may not indicate that runaway failure would occur sooner.

Model $V_4$ requires larger displacements for tensile membrane action to be mobilised, as its tensile capacity is entirely dependent on the reinforcement. From Fig. 5.15, it is observed that the Bailey-BRE method predicts a higher capacity than its comparable finite element analysis (the $V_4$ model). This discrepancy shows that the method is unconservative. Also, in practice, the real perimeter beams will achieve temperatures higher than 20°C, and will experience vertical displacement due to their applied loads and loss of strength.

![Graph showing comparison of deflections](image)

**Fig. 5.15: Comparison of BRE deflection and *Vulcan* $V_3$ and $V_4$.**

### 5.3.4.3 Effect of continuity at the panel boundary

The effect of slab panel continuity on the development of tensile membrane action was also investigated. Fig. 5.16 shows one quarter of a typical composite floor divided into slab panels of dimensions 9.0m x 7.5m. To isolate the effect of rotational restraint at the panel boundaries the panels were analysed as being independent, with no axial restraint but rotational restraint about their edges where adjacent slabs were present. Protected beam temperature profiles were the same as in the *Vulcan* $V_1$ and $V_2$ models.

![Graph showing vertical displacements](image)

**Fig. 5.17: Comparison of central vertical displacements.**

The results indicate that continuity with adjacent slabs helps to maintain some level of vertical support, as the protected
perimeter beams are not allowed to rotate about their axes. The *Vulcan* $V_5$ model, which had rotational restraints about all four edges, is comparable, at 60 minutes with a deflection of 367mm, to the *Vulcan* $V_2$ model with vertical supports along its entire perimeter. Further, Fig. 5.18 shows a comparison of the results in Fig. 5.17 with the Bailey-BRE displacement.

The results of Figs. 5.17 and 5.18 show that the $V_5$ slab panel is aided by the presence of hogging moments at its ends. From the results, it is observed that the edge and corner panels fail quicker than the internal panels. This is because their degrading simply supported edges cannot bear the imposed loads indefinitely, and require some strengthening to have a comparable effect to their internal edges.

![Fig. 5.16: Continuous slab panels – layout for Fig. 5.17](image)

![Fig. 5.17: *Vulcan* analyses with rotational restraints and generic protection](image)
It is also observed that panels with more rotational restraint across their primary beams performed better than those with more rotational restraint on their secondary beams (comparison of *Vulcan* models V6 and V7).

It may be argued that in reality there will be anchorage of the reinforcing bars across the perimeter beams and, as such, higher capacities will be obtained. However, the presence of axial restraint would cause higher deflections in the initial stages of fire exposure, because of thermal buckling due to restrained thermal expansion, and the appropriateness of the Bailey-BRE prediction would be called into question.

### 5.3.4.4 Thickness of the protection material

Fig. 5.19 shows a comparison of the Bailey-BRE deflection, *Vulcan* model V1 and *Vulcan* model V9, which had twice the required thickness of generic protection on the protected boundary beams. It can be observed that the Bailey method suggests higher capacity than the V9 model, although its protected beams are much colder than V1. The analysis confirms the unconservative nature of the Bailey approach, which suggests that vertical restraint may be achieved by the use of heavier sections or thicker protection, which may be costly.
5.3.4.5 Effect of reinforcement

The Bailey-BRE method depends heavily on reinforcement ratios. The dependence of the method on yield-line capacities suggests that increasing reinforcement ratios can increase the fire resistance of composite slab panels in fire. Therefore, further investigation was carried out with larger meshes on the 7.5m x 9m slab panel. The additional mesh sizes used were A252 and A393. Only the critical *Vulcan V*₁ model was tested for this effect.

Fig. 5.20 shows displacement-time plots of the *Vulcan V*₁ model with reinforcement mesh sizes A193, A252 and A393. It can be seen that, although increasing the reinforcement mesh size does increase the capacity of the slab panel system, the reinforcement does not affect the slab panel ultimate failure time or mode in proportion to the extra areas of steel used; even the displacement-based slab panel capacity does not increase.
Failure of the slab panel is due to the failure of the protected secondary beams, as explained in Section 5.3.4.1. Section 5.4 therefore details a proposed failure mechanism for slab panel boundary beams in fire.

5.4 Edge support failure mechanism

5.4.1 Slab panel behaviour

The preceding section examined the effects of various parameters on the failure of slab panels in fire. The observations indicate that the effects of boundary edge support and protected beam temperature cannot be ignored when a simple design method like the Bailey-BRE method is concerned.

The results show that tensile membrane action is maintained up to a point where the combined effects of the increasing load on the protected beams and their thermal degradation cause failure of the protected beams. The results also show that additional support along any of the panel boundaries improves slab capacity. To assess the decline of vertical support in the Bailey-BRE (TSLAB) approach, a simple edge support failure mechanism has been proposed.
Chapter 5: Slab panel vertical support

5.4.2 Proposed failure mechanism

5.4.2.1 Derivation

The mechanism considers the failure of parallel arrangements of either primary or secondary beams. The folding mechanism which is observed developing in Figs. 5.11 to 5.14 is explained in Fig. 5.21. In this figure, the intermediate unprotected beams are shown with thick dashed lines, with the protected beams shown in solid continuous lines. Yield lines are shown as dotted lines.

After ambient temperature loading, the intermediate unprotected beams within the panel start losing strength. When all their strength is lost, plastic hinges form and their loads are borne by the slab in tensile membrane action, supported on the protected edge beams. With time, the protected edge beams lose strength and experience more deflections with the loads from the slab. At that stage, the central yield-line extends towards the weaker system of parallel edge beams. This behaviour continues until a time when the formation of plastic hinges in the protected beams cause collapse of the slab panel.

Depending on the capacities of the protected primary and secondary beams the two modes of failure are subsequently derived.

In the derivations, the following notations are used:

- $L$: length of the primary beam
- $l$: length of the secondary beam
- $w$: applied floor load at the fire limit state
- $δ$: deflection of the slab panel (or beams)
- $θ$: beam rotation
- $M_{pp}$: plastic moment capacity of the protected primary beam at time $t$
- $M_{ps}$: plastic moment capacity of the protected secondary beam at time $t$
- $M_{u}$: plastic moment capacity of the unprotected beam at time $t$
- $n$: number of intermediate unprotected beams in the slab panel
Fig. 5.21: Failure mechanism of protected edge beams

Failure of Secondary beams

\[ \theta = \frac{2\delta}{l} \]

Since there are 2 protected beams and a number of unprotected secondary beams within the panel, the internal work done is calculated as:
\[ \text{Internal work done} = 2\left(2 \times 2M_{ps} \theta + 2nM_u \theta \right) \] (5.2)

The external work done is:
\[ \text{External work done} = wV = wL\delta \] (5.3)

Therefore, for failure to occur:
\[ wL\delta - \left(8M_{ps} \theta + 4nM_u \theta \right) \geq 0 \]
\[ wL\delta - \left(\frac{16M_{ps}}{l} + \frac{8nM_u}{l}\right) \delta \geq 0 \]
\[ wL^2 - 8\left(2M_{ps} + nM_u\right) \geq 0 \]
\[ \frac{wL^2}{8} - \left(2M_{ps} + nM_u\right) \geq 0 \] (5.4)

**Failure of Primary beams**

\[ \theta = \frac{2\delta}{L} \]

Since there are only 2 protected primary beams, the internal work done is calculated as:
\[ \text{Internal work done} = 2\left(2 \times 2M_{pp} \theta \right) \] (5.5)

The external work done remains the same as Equation 5.3. Therefore, for failure to occur:
\[ wL\delta - 8M_{pp} \theta \geq 0 \]
\[ wL\delta - \left(\frac{16M_{pp}}{L}\right) \delta \geq 0 \]
\[ wL^2 - 8\left(2M_{pp}\right) \geq 0 \]
\[ \frac{wL^2}{8} - 2M_{pp} \geq 0 \] (5.6)
5.4.2.2 Verification

The proposed failure mechanism was tested on the 7.5m x 9m slab panel. Using Equations 5.4 and 5.6 at various time steps, it was discovered that failure of the protected secondary beams (hence failure of the entire slab panel) occurred at 70 minutes. This is highlighted in Fig. 5.21 by the vertical black line. Failure of the primary beams was not observed within the 90-minute exposure.

![Graph showing vertical displacement over time](image)

**Fig. 5.22: Plastic failure of protected edge beams**

The observation of secondary beam failure above confirms the visual *Vulcan* output in Figs. 5.11 to 5.14. The bottom flange temperatures of the unprotected intermediate and protected secondary beams were 961°C and 597°C respectively.

In the 7.5m x 9m slab panel, failure of the protected primary beams was not observed. Therefore, three additional slab panels were investigated in *Vulcan* with differing reinforcement, to see if significant differences were observed with different aspect ratios and also check the modes of failure of the slab panels.

The panels were 9m x 6m, 9m x 9m and 9m x 12m, as shown in Fig. 5.23. They were all designed for a 60 minute fire resistance with exposure to the Standard Fire. Their thermal and physical properties were the same as those of the 7.5m x 9m slab panel, except that they were designed for an imposed load of 5.0kN/m² instead of the 3.5kN/m² in the 7.5m x 9m model. Tables of the design requirements and their protected beam properties, using lightweight fire-resisting gypsum boards, are shown in Tables 5.4 and 5.5. To adequately observe failure of the protected beams, the load...
ratios of the protected beams were kept within the range 0.4 to 0.5, with the primary beams having slightly higher values.

![Diagram showing slab panel sizes and support types](image)

**Fig. 5.23: Additional slab panel sizes**

<table>
<thead>
<tr>
<th>Slab Panel size</th>
<th>9m x 6m</th>
<th>9m x 9m</th>
<th>9m x 12m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead load (kN/m²)</td>
<td>4.33</td>
<td>4.33</td>
<td>4.33</td>
</tr>
<tr>
<td>Live load (kN/m²)</td>
<td>5.0</td>
<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Additional load (kN)</td>
<td>14</td>
<td>37</td>
<td>49</td>
</tr>
<tr>
<td>Beam design factor</td>
<td>0.77</td>
<td>1.00</td>
<td>0.83</td>
</tr>
<tr>
<td>Min. Mesh size</td>
<td>A193</td>
<td>A193</td>
<td>A252</td>
</tr>
</tbody>
</table>

**Table 5.4: Slab panel design requirements**

Beam temperature distributions are shown in Fig. 5.24. Again, TSLAB unprotected beam temperatures were used in the *Vulcan* models. Only the V₁ type of *Vulcan* analysis (with generic protection on all edge beams and corner vertical support) was conducted.
In Fig. 5.24, temperatures for unprotected intermediate beams are shown as dark blue lines, while those for protected primary and secondary beams are shown as blue and brown lines respectively. The Standard Fire curve is shown in red.

The results of the *Vulcan* analyses are presented in Figs. 5.25 to 5.27. Fig. 5.25 shows results for the 9m x 6m slab panel while those for the 9m x 9m and 9m x 12 slab panels are shown by Figs. 5.26 and 5.27, respectively. The figures show results from a *Vulcan* $V_1$ type analysis.
Results using A193 mesh are shown in red, while those with the A252 and A393 meshes are shown in green and black respectively. The failure time for each slab panel is indicated by the vertical black line. The Bailey-BRE absolute vertical deflection limit and the short span/20 deflection criterion (shown as the horizontal pink and blue lines, respectively) are also drawn on the graphs for comparison. For each panel, the displacements of the centre of the slab panel with respect to the midpoints of the protected secondary and primary beams are also shown. The slab panel central displacements relative to the midpoint of the secondary beam are shown with short dashes, and those relative to the midpoint of the primary beams are shown in dotted lines.

From the results it is observed that each of the slab panels eventually collapses by failure of the protected secondary beams, with a marked decrease in relative deflections towards failure. Failure times and temperatures of the unprotected intermediate and protected secondary beams are shown in Table 5.6. It should be noted that primary beam failure was not observed within 90 minutes of fire exposure.

<table>
<thead>
<tr>
<th>Slab panel</th>
<th>Failure Time (mins)</th>
<th>Intermediate beam temperature (°C)</th>
<th>Secondary beam temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9m x 6m</td>
<td>82</td>
<td>983</td>
<td>663</td>
</tr>
<tr>
<td>9m x 9m</td>
<td>73</td>
<td>963</td>
<td>621</td>
</tr>
<tr>
<td>9m x 12m</td>
<td>68</td>
<td>952</td>
<td>594</td>
</tr>
</tbody>
</table>

Table 5.6: Slab panel failure times and beam temperatures

It is observed that all the slab panels met their 60-minutes design criterion. However, they were designed so that their protected beam temperatures reached 550°C at 60 minutes, while in practice this value is usually 620°C. This indicates that these slab panels could potentially fail if they were placed on the façade of a building with edge beam design temperatures of 620°C at 60 minutes.
Fig. 5.25: Vertical displacements of the 9m x 6m slab panel

Fig. 5.26: Vertical displacements of the 9m x 9m slab panel
5.5 Collapse mechanisms

A collapse mechanism of parallel arrangements of primary or secondary beams has been presented in Section 5.4. In the series of simply supported slab panels that were examined, the folding mechanism occurred across secondary beams. It was observed that the tensile membrane capacity of the slab panel depends on support from the protected beams, which in-turn depends on the beam temperature and its applied load. The loss of edge support leads to single-curvature bending; this may either cause a runaway folding failure of the slab, or induce catenary action, which may fracture connections.

In practice, slab panels are usually continuous over at least two supports. As observed in Section 5.3.4, continuity provides higher slab panel resistance in fire. However, depending on the extent of the fire in a building and the lightness of the reinforcement used in composite floor construction, the continuity may be lost, or significantly higher loads may be imposed on the protected perimeter beam between two adjacent slab panels. Coupled with thermal degradations, these beams can experience fairly large deflections, and may collapse. Therefore, it is prudent to examine possible collapse mechanisms which can develop in these slab panels, so that they can be monitored in designs that employ the Bailey-BRE method, in order to ensure that each panel can develop its full tensile membrane capacity and not fail by the losing support from the
protected beams. An examination of all possible scenarios offers the possibility of selecting the mechanism which requires the least plastic energy. A discussion of possible collapse scenarios is therefore provided in this section. For simplicity, and in order to be consistent with the assumptions of the Bailey-BRE method, only isolated slab panels are considered.

5.5.1 Collapse mechanism 1
Fig. 5.28 shows a possible collapse mechanism that can occur in corner compartments of a composite steel-framed building. For design, the panels are isolated, as required by the Bailey-BRE method. The loss of strength and stiffness of the intermediate unprotected beams results in the formation of plastic hinges, which causes a redistribution of floor loads to the edge beams.

![Diagram showing collapse mechanism 1](image)

The increased loads and material degradation of the protected secondary and primary edge beams may cause yielding. With the edge beams folding, the column at the corner of the building is pulled inwards. This behaviour can accelerate buckling of the column due to p-delta effects. Also, the accelerated deflections of the edge beams cause the rest of the slab to hang from the two other protected edges, which may be experiencing some deformations due to thermal exposure, causing catenary forces in the adjacent slabs. The mechanism described above presumes failure of the edge beams, but can occur in any two adjacent beams, depending on their lateral support conditions and the loads on the members. The four possible scenarios are shown in Fig. 5.28 above.
5.5.2 Collapse mechanism 2
The second folding mechanism is shown in Fig. 5.29. In this mechanism, the formation of a plastic hinge in one of the protected edge beams initiates folding of the slab panel along the diagonal fold lines. This collapse mechanism is most likely to occur in a panel on the edge of a building, where the edge beams on the façade lack rotational restraint, or in cases where the interior beams are heavily loaded, with rapidly deteriorating materials.

![Diagram of collapse mechanism 2](image)

Fig. 5.29: Collapse mechanism 2, showing catenary forces

The lack of continuous vertical support along the boundary implies that the yield lines, developed through the loss of strength of the intermediate beams, will propagate towards the weakest edge support, which could be on a primary or secondary beam, depending on the geometry of the panel, the imposed loads and the temperatures of the individual protected beams. The four possible failure scenarios and the potential catenary forces of this collapse mechanism are shown in Fig. 5.29.

5.5.3 Collapse mechanism 3
The third collapse mechanism has already been discussed in Section 5.4. The two alternatives are:

1. Folding across the parallel arrangement of primary beams
2. Folding across the parallel arrangement of secondary beams (protected and unprotected)
Earlier sections of the chapter showed that the Bailey-BRE method ignores the potential slab panel failure that may occur when edge beams experience large deflections and fold plastically due to increased loads and material degradation. The present section has identified two other possible collapse mechanisms which can hasten the folding failure of the floor system. It is believed that incorporating the complex failure mechanisms outlined in this section, and the simpler one in Section 5.4, into the Bailey-BRE method will help ensure that the loss of tensile membrane capacity due to edge beam failure is eliminated.

5.6 Summary
A number of protection schemes and support conditions of slab panels have been analysed. It has been observed that the tensile membrane action mechanism is lost when slab panel edge beams experience significant deflections and create plastic hinges. Considerable restraint is provided by either vertically supported edges or continuous slab panels. The results show that the Bailey-BRE method gives a good prediction of slab panel behaviour if the perimeter beams continue to provide vertical support for long periods of time.

A plastic failure mechanism for slab panels indicating plastic collapse of the edge beams has been proposed. It determines failure of the slab panel by examining the reduction of composite beam capacity with increasing thermal exposure. The analyses have shown that a combination of the imposed load and material degradation will cause failure. Therefore, specifying a protected beam temperature of $620^\circ C$ at the required fire resistance time is not necessarily sufficient. It has also been observed that failure of the parallel arrangement of secondary beams will usually precede failure of primary beams. For slabs in the interior of a building, the restraint from adjacent slabs is clearly beneficial, but for edge or corner slab panels increasing the level of protection seems a viable option. This could potentially counter the reduction in cost given by employing tensile membrane action. However, producing safe structures should be a priority over economy.

A discussion of other possible folding mechanisms in composite floor slabs is also given. It shows that potential structural failure scenarios can develop if the design of the slab panels does not adequately consider the ability of the protected edge beams to provide vertical support.
The Bailey-BRE method serves as a good predictor of slab panel behaviour in fire. The study in this chapter has shown that it is ideal for interior panels, but requires an edge beam failure criterion where slab panels on the façades of buildings are concerned. It also requires consideration of typical folding mechanisms to ensure that tensile membrane action can be sustained by the panel within its required fire resistance time.
Chapter 6: Effects of reinforcement ratios

6 Effects of Reinforcement Ratios

Composite slabs are normally constructed to act as the compression flanges of a series of composite beams. To prevent cracking during construction, these slabs are reinforced with light mesh, of sizes between 142mm$^2$/m and 393mm$^2$/m. At large deflections or in fire, tensile membrane action of the composite slabs increases the capacity of composite slabs, if a structure is designed to allow for this mechanism. At very high temperatures and deflections, the tensile capacity of a slab panel will depend on the area of reinforcement in the composite slab. However, it has been observed that increasing the mesh sizes in composite slab panels does not proportionately increase their resistance. The discrepancy may be due to the geometry, composition or support conditions of the slab panels. Chapter 5 examined an aspect of this discrepancy by investigating the influence of various support conditions on the mobilisation and sustenance of tensile membrane action. This chapter therefore explores possible limitations of the Bailey-BRE-TSLAB membrane action method with reference to the geometry and the area of reinforcement in the slab panel.

The chapter attempts to establish the range of applicability of the membrane action method, and suggests solutions for those areas in which problems are found. A comprehensive review of some of the basic assumptions of the method is made, and its failure criterion is also reviewed. With these, the shortcomings of the method in defining the limit of its slab capacity design in reference to reinforcement sizes are identified.

6.1 Research on reinforcement ratios

6.1.1 Finite Element Analyses with Vulcan

A limited number of previous studies have compared the Bailey-BRE membrane action method with more fundamental approaches based on finite element analysis. These have highlighted a number of discrepancies between the two approaches. One which has attracted particular interest is the effect of increasing slab reinforcement ratios.

In modelling the British Steel corner test of Cardington, Huang et al. (2001a) investigated the effect of the areas of reinforcement meshes on the performance of the 9m x 6m composite floor. Half and twice the size of A142 mesh were used, in addition to the standard mesh, for the study. It was observed that, after the unprotected steel
beams had reached 500°C, the tensile forces in the centre of the slab were mainly carried by the reinforcement, and an increase in the size of the mesh increased the capacity of the composite floor. Further research by these authors using standard A142, A193, A252 and A393 meshes revealed that, with a higher reinforcement mesh size, and a reasonable fire protection regime for protecting beams on the main gridlines while leaving intermediate secondary beams unprotected, slab panels could be designed adequately for 30- or 60- minute fire resistance on the basis of BS5950: Part 8 (BSI, 2003) limiting temperature requirements (Huang et al., 2001b).

6.1.2 Observed discrepancies
As mentioned in Section 5.1, Huang et al. (2001a; 2002a; 2004b) compared the Bailey-BRE method and the finite element software Vulcan. Observed differences ranged from the horizontal edge-support conditions to the effects of increasing reinforcement ratios on slab panel capacity, considering slab panels of aspect ratios 1.0 and 2.0. It was observed that increasing the reinforcement mesh size for the slab with the lower aspect ratio showed a disproportionate increase in slab panel resistance given by the Bailey-BRE method, while only marginal increases were observed with the finite element method. To isolate the effect of strength loss of the protected edge beams, the analysis with the higher mesh size was re-run with increased protection for the protected beams. However, a similar observation was made.

The two methods (Vulcan and Bailey-BRE) however compared very closely when the slab panel aspect ratio was increased to 2.0 and the higher reinforcement mesh of A393 was used. It was also observed that the behaviour of this panel was primarily due to catenary action rather than tensile membrane action.

6.1.3 Experimental research on reinforcement ratios
The above observation led to a number of experiments on this subject. Experimental tests on small-scale slabs at ambient and elevated temperatures were performed by Foster (2006) to investigate the tensile membrane action phenomenon in thin concrete slabs reinforced with various reinforcement mesh sizes and textures. These experiments were supplemented by the present author (see Chapter 2) and by Bailey and Toh (2007a; 2007b). These experimental investigations confirmed the disproportionateness of the increase in slab panel capacity when the Bailey-BRE method is used with higher reinforcement. They also showed that higher reinforcement ratios change the failure mechanism of thin concrete slabs from tensile fracture of
reinforcement in the middle regions of the slab to compressive crushing of concrete at the corners.

This experimental confirmation of the numerical analyses of Huang et al. (2001a; 2002a; 2004b) led Bailey and Toh (2007a; 2007b) to define a compressive failure criterion in a revision to the Bailey-BRE membrane action method. Compressive failure is initiated when reinforcement areas exceed a critical level, compared to the available compression capacity of the concrete. The new method was used to evaluate the test results of Bailey and Toh (2007a; 2007b). The tests, however, showed that compressive failure was only observed at ambient temperature, and never at elevated temperatures. Slab panel analyses with TSLAB 2.4 (the SCI Microsoft Excel implementation of the Bailey-BRE design method) still show increased slab panel capacities with higher reinforcement ratios, the enhancement depending on the aspect ratio selected.

The preceding chapter has shown that the Bailey-BRE enhancement of capacity due to deflection gives good predictions when continuity in slabs is considered, as the non-conservative assumption of vertically supported boundaries is aided by the rotational restraint along the panel edges. It was shown in Chapter 5 that loss of stiffness of the beams on the perimeter may cause a collapse of the slab panel, regardless of the size of its reinforcement, as the slab panel loses its tensile membrane action mechanism, and effectively hangs in catenary from its corners.

Research so far into the effects of reinforcement on concrete slabs in fire has focused on slabs with continuous vertical support along their edges and hinge-supported beams (Huang et al.; 2002a; 2004b; Foster, 2006; Bailey and Toh, 2007a; 2007b). However, the Bailey-BRE method was designed for simply-supported slab panels, and its minimum reinforcement ratios as defined in tables in the SCI P-288 document (Newman et al., 2006) are based on the assumption of simply-supported edges which allow horizontal pull-in.

As observed in the previous chapter, and various results from tests, corner bays are susceptible to collapse by failure of protected secondary beams; these slab panels may not have sufficient horizontal or rotational restraint along their free edges, and will rely on the area of reinforcement to provide the requisite tensile membrane capacity in fire. The impetus of this chapter is therefore to extend the research in Chapter 5 to investigate the effects of reinforcement sizes on the failure of slab panels in fire, in
relation to the geometries of the slab panels, by using finite element approaches and comparing the results to the Bailey-BRE method.

6.2 Review of Bailey-BRE method

Bailey and Toh (2007a) have recently updated the Bailey-BRE membrane action method. Two in-plane stress patterns are now used in predicting the tensile fracture of the reinforcement in the central zone of the slab panel. A compressive failure criterion has also been added to account for failures with higher reinforcement ratios.

6.2.1 In-plane stress patterns

The revised in-plane stress patterns help to provide more precise stress distributions. In the original method (Bailey, 2000; 2001; 2003), it was assumed that, at failure, the compressive ring around the periphery of the slab produced a resultant compressive force concentrated at the edge of the slab. This is used in the determination of the parameter $b$, which defines the in-plane membrane force:

$$b = \frac{1.1 l^2}{8K(A + B + C - D)} \quad (6.1)$$

where $A$, $B$, $C$, $D$, $K$ and $l$ are as defined in the references (Bailey, 2000; 2001). The 1.1 factor in the equation above implements the assumption that this is the ratio of the reinforcement’s ultimate strength to its yield strength.

In the revised version of the method, triangular stress distributions along the diagonal yield lines are replaced by trapezoidal distributions (Fig. 1.8). The $b$ parameter is given by one of two values, depending on the geometry of the slab panel, the thickness of the concrete slab and the ultimate strengths of the reinforcement in the two directions. For stress pattern 1, $b$ is:

$$b = \frac{1}{K(A + B)} \left(1 - 2m \right) \left(1 + 2 f_2 + \frac{2m}{3} \right) \leq f_1 \quad (6.2)$$

In the above equation, $f_1$ is the ratio of the ultimate strength of the reinforcement to its yield strength in the shorter span. If, however, the value of $b$ is found to be greater than $f_1$, then the alternative stress pattern results, and the value of $b$ is equal to $f_1$. The parameters in Equation 6.2 above are defined in the reference (Bailey and Toh, 2007a).
6.2.2 Compressive failure

The compressive failure criterion is defined by equating the sum of the in-plane and bending compression stresses to the ultimate compressive capacity of the concrete slab. It is assumed that the maximum compressive stress block depth is limited to \(0.45d\), where \(d\) is the average reinforcement mesh depth. Therefore, the \(b\)-parameter for determination of membrane capacity is derived using Equation 6.3 (Bailey and Toh, 2007a).

\[
b = \frac{1}{kK_T} \left[ 0.67 f_{cu} \times 0.45 \left( \frac{d_1 + d_2}{2} \right) - T_0 \left( \frac{K + 1}{2} \right) \right]
\]  

(6.3)

6.2.3 Comparison of old and new Bailey-BRE methods

A comparison of the old and new Bailey-BRE methods is given here. A 7.5m x 7.5 m slab panel has been analysed for 60 minutes fire resistance. The panel has two unprotected S275 305x127x33UB section beams within it, spaced at 2.5m. A 130mm deep trapezoidal concrete slab profile with cube strength of 35N/mm² is used. The temperature profiles of Figs. 5.5 and 5.6 are used, with the average depth of reinforcement maintained at 45mm. An imposed load of 5.0kN/m² is applied to the normal-weight concrete slab, with dead loads of 4.33kN/m². In all, four A-mesh-type reinforcement sizes were used. This particular slab panel was selected to highlight the effect of using higher reinforcement ratios, as shown in the new Bailey-BRE method. The reinforcement sizes were 142, 193, 252 and 393mm²/m. Results are shown in Table 6.1 and Fig. 6.1.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Old method</th>
<th>New method</th>
<th>New method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(P) (kN/m²)</td>
<td>(b)</td>
<td>(e)</td>
</tr>
<tr>
<td>A142</td>
<td>1.31</td>
<td>1.594</td>
<td>4.41</td>
</tr>
<tr>
<td>A193</td>
<td>1.76</td>
<td>1.564</td>
<td>3.29</td>
</tr>
<tr>
<td>A252</td>
<td>2.27</td>
<td>1.574</td>
<td>2.56</td>
</tr>
<tr>
<td>A393</td>
<td>3.40</td>
<td>1.552</td>
<td>1.70</td>
</tr>
</tbody>
</table>

Table 6.1: Comparable parameters of the old and new Bailey-BRE methods

Table 6.1 shows comparable parameters in the old and new Bailey-BRE methods. \(P\) is the yield-line capacity, \(e\) is the enhancement required by the concrete slab to sustain its share of the applied load (5.80kN/m²) at 60 minutes, after the unprotected composite beams have lost considerable strength. The \(b\) values calculated by the two methods considering the various reinforcement mesh sizes are as shown in Table 6.1. For the new method, \(b_1\) is calculated assuming failure as tensile fracture of reinforcement across the short span while \(b_2\) is calculated assuming that failure is controlled by
crushing of concrete at the corners of the slab panel. The method uses the minimum \( b \) value in determining the required deflection.

Fig. 6.1 shows the necessary deflections of the two methods. The solid curves show the results of the new Bailey-BRE method (Bailey and Toh, 2007a) while the old method (Bailey, 2000; 2001; 2003) is shown in dashed lines. Results for A142, A193, A252 and A393 meshes are shown in blue, red, green and black, respectively. From Fig. 6.1, it is observed that the two methods are comparable except in the case of the A393 reinforcement. Upon investigation (and as shown in Table 6.1), it was found that failure of the slab panel reinforced with the A393 mesh in the new method was compressive in nature.

![Fig. 6.1: Comparison of the old and new Bailey-BRE methods](image)

The higher deflections given by the new method for higher reinforcement areas partly addresses a major concern raised by Huang et al. (2002a; 2004b) that the old method showed rapidly increasing slab capacities with increased reinforcement. The inclusion of compressive failure in the method limits this increased capacity.

### 6.2.4 Other comparisons

Subsequent to the development of the new method Toh and Bailey (2007) compared it with TSLAB and Vulcan. The research examined a number of long-span slab panels with intermediate and secondary beam spans ranging from 14m to 18m while primary
beams ranged from 6m to 12m. The total slab depth was 130mm. S355 steel was used for all beams, with the protected beams reaching 550°C at 90mins.

The comparison of the improved Bailey-BRE method and TSLAB showed that the method predicted higher allowable vertical displacements, higher enhancement factors and therefore higher slab load capacities than the TSLAB approach. It was also observed that enhancement factors did not always increase with decreasing aspect ratios in the TSLAB approach, implying that the method was not applied exactly in the TSLAB program. Only one slab panel (14m x 12m) generated enough capacity (9.19kN/m²) to support the fire limit state load of 8.00kN/m². The Vulcan results did not show any signs of failure, even after 90 minutes of exposure to the Standard temperature-time curve, although large deflections of the slab panels were observed. The finite element analyses also showed that slab panels with intermediate and secondary beam spans of 14m to 16m and aspect ratios not exceeding 1.56 satisfied the Bailey-BRE deflection limit at 90 minutes. The paper, however, does not describe the finite element analyses in detail, and therefore does not paint a clear picture of the support conditions of each slab panel in the finite element analyses. As observed from Chapter 5, support conditions play a crucial role in the formation and sustenance of tensile membrane action.

6.2.5 Present study
This chapter therefore performs a more extensive comparison of the Bailey-BRE method and Vulcan. The study aims to identify the areas where the assumptions inherent in the method make it suitable for the design of slab panels, by examining different aspect ratios and reinforcement sizes. It has already been established in Section 6.2.3 that higher reinforcement areas could influence the failure of the slab panel as presented by the Bailey-BRE method. Therefore, this chapter examines the likelihood of failure by corner crushing, and investigates the relationship between slab panel sizes and the minimum reinforcement size required for a slab panel for a particular fire resistance.

6.3 Comparative Study of Vulcan and Bailey-BRE Method
To cover a reasonable range of analyses the three slab panel sizes of Fig. 5.23 were used for the study. The 9m x 6m, 9m x 9m and 9m x 12 panels were designed for 60 minutes’ fire resistance, assuming normal-weight concrete and imposed loads of 5.0kN/m² plus 1.7kN/m² for ceilings and services. Using guidance from the SCI-P288
document (Newman et al., 2006), reinforcement mesh sizes of A142 to A393 with a strength of 500N/mm² were chosen for the investigation. The slab temperature profile of Fig. 5.5 was used in this series of analyses. To predict adequately the influence of reinforcement size on failure, the Vulcan analyses model the protected edge beams, with full vertical support only provided at each corner of the panels, as explained in Section 5.3.3. The explicit assumption of vertically supported slab panel edges in the Bailey-BRE approach is maintained for comparison when using this method.

It has been necessary to assume failure criteria for both models. These are; the TSLAB limiting deflection, the maximum allowable deflection limit of the BRE method, and the short span/20 criterion. The Bailey method is based on a calculation of the deflection required to enhance the slab’s small-deflection yield-line capacity, comparing this with a limiting deflection based partly on test observations. There is a difference between deflection limits in TSLAB and the original Bailey-BRE method (Toh and Bailey, 2007, see Section 1.4.2). TSLAB uses Equation 5.1, while in the Bailey-BRE method the values of \(f_y\) and \(E\) change with reinforcement temperature at each time step. In this chapter, therefore, Equation 5.1 is used as one of the deflection limits, because the difference in the two limits is more pronounced when reinforcement strengths exceed 400°C; the reinforcement temperatures shown in Fig. 5.5, however, do not exceed 400°C. The required Bailey-BRE deflections shown here are based on the improved method as presented by Bailey and Toh (2007a).

Following BS 5950 part 3 (BSI, 1990) and BS 5950 part 8 (BSI, 2003) the beam sections in Table 6.2 were obtained. Protected beams were designed to attain temperatures just below 550°C at 60 minutes, using lightweight fire-resisting gypsum boards (as in Section 5.3). Again, unprotected intermediate beam temperatures in the Vulcan models were obtained from the TSLAB program, for proper comparison. The beam temperatures for the different slab panel sizes are shown in Fig. 5.24.

<table>
<thead>
<tr>
<th>Slab Panel</th>
<th>Beam Type</th>
<th>Beam Section</th>
<th>Load ratio</th>
<th>Limiting temperature</th>
<th>R60 Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>9m x 6m</td>
<td>Intermediate</td>
<td>356x171x57</td>
<td>0.370</td>
<td>657°C</td>
<td>931°C</td>
</tr>
<tr>
<td></td>
<td>Secondary</td>
<td>356x171x57</td>
<td>0.426</td>
<td>636°C</td>
<td>548°C</td>
</tr>
<tr>
<td></td>
<td>Primary</td>
<td>406x178x60</td>
<td>0.452</td>
<td>627°C</td>
<td>549°C</td>
</tr>
<tr>
<td>9m x 9m</td>
<td>Intermediate</td>
<td>305x127x48</td>
<td>0.470</td>
<td>621°C</td>
<td>923°C</td>
</tr>
<tr>
<td></td>
<td>Secondary</td>
<td>356x171x67</td>
<td>0.442</td>
<td>630°C</td>
<td>550°C</td>
</tr>
<tr>
<td></td>
<td>Primary</td>
<td>533x210x101</td>
<td>0.446</td>
<td>629°C</td>
<td>548°C</td>
</tr>
<tr>
<td>9m x 12m</td>
<td>Intermediate</td>
<td>356x171x57</td>
<td>0.370</td>
<td>657°C</td>
<td>931°C</td>
</tr>
<tr>
<td></td>
<td>Secondary</td>
<td>406x178x67</td>
<td>0.447</td>
<td>629°C</td>
<td>548°C</td>
</tr>
<tr>
<td></td>
<td>Primary</td>
<td>610x305x179</td>
<td>0.471</td>
<td>620°C</td>
<td>547°C</td>
</tr>
</tbody>
</table>

Table 6.2: Beam design data for the slab panels (R60)
6.3.1 Structural analyses

The study of the 7.5m x 9m slab panel in the previous chapter examined various support conditions which provided the necessary vertical support for tensile membrane action to occur. At that stage only one reinforcement mesh size (A193) was used, in order to observe adequately the difference in failure times attributable to the support conditions alone. Further analyses conducted with A252 and A393 meshes showed that these did not change the panel failure time, because failure was due to the loss of strength and stiffness of the protected secondary beams. The subsequent study, reported in this chapter, is therefore designed to follow up slab panel failure with respect to the effects of reinforcement and slab panel geometry.

The structural analyses in this section follow the pattern developed in Chapter 5. They concern simply-supported slab panels. Finite element models are analysed with Vulcan, and compared with the Bailey-BRE method. The Vulcan analyses allow vertical displacement of the protected beams on the panel perimeter. An objective of the analyses is to compare the finite element analyses to those of TSLAB, but Toh and Bailey (2007) have found inconsistencies between TSLAB and the general Bailey-BRE method, so both the original Bailey-BRE equations and the TSLAB limiting deflection are used in the investigation.

6.3.1.1 Results and Discussions

Comparisons of deflections given by the Vulcan and Bailey-BRE models are presented in this section. The Vulcan and Bailey-BRE failure times and temperatures are compared using the TSLAB limiting deflection curve (shown as solid pink), the BRE maximum allowable deflection (as a dashed pink line) and the span/20 deflection (in blue). For clarity, the deflections in this section are also colour-coded. Results from the two models are distinguished by different types of line.

These are:

**Deflection plots for given:**
- A142 – Navy blue
- A193 – Red
- A252 – Green
- A393 – Black

**Analytical approaches:**
- BRE method - Solid lines
- Vulcan - Broken lines

It should be noted that, unless otherwise stated, the results presented in this section show absolute maximum vertical displacements of the middle of the slab panel.
**9m x 6m Slab Panel**

Fig. 6.2 shows the deflections required to enhance the capacity of the 9m x 6m slab in the Bailey-BRE method. It is observed that, depending on the choice of deflection criterion and reinforcement mesh size, different slab panel capacities are obtained. It is striking that increasing the reinforcement area from A193 to A252 increases the slab panel survival time from 37 minutes to over 90 minutes using the TSLAB criterion and from 34 minutes to about 85 minutes with the span/20 criterion. It should be noted that SCI P288 (Newman *et al.*, 2006) specifies A193 as the minimum reinforcement mesh size required to generate sufficient slab panel capacity.

![Graph showing Bailey-BRE Results – 9m x 6m Slab Panel (R60)](image)

In Fig. 6.3, a comparison of the design method with *Vulcan* predictions is shown. The results show that the *Vulcan* slab panel's survival times hardly vary with increasing reinforcement mesh size, due to the failure of the protected secondary beams, indicated by the vertical black line. *Vulcan* panels reinforced with A142 and A193 meshes show higher resistances compared with their Bailey-BRE models. The reverse is observed for A252 and A393.

Typically, designers would specify a span/20 deflection of 300mm for a slab panel of this geometry in fire. With this deflection criterion, the A252-reinforced Bailey-BRE model is adequate, but all *Vulcan* models do not meet the required 60-minute fire resistance, although all protected beams are at 550°C. For *Vulcan*, the results show that run-away failure occurs at 82 minutes (Fig. 6.3); with protected secondary beam temperatures at 663°C.
Chapter 6: Effects of reinforcement ratios

Fig. 6.3: Comparison of Results and Edge beam failure – 9m x 6m Slab Panel (R60)

It can be argued that the edge beams in the Bailey-BRE method are assumed to stay vertical, and therefore represent relative deflections. Relative displacement plots in Chapter 5 established that failure of the slab panel occurred by the formation of plastic hinges in the protected secondary beams. A comparison of the relative displacements of the slab with respect to the secondary beams is therefore shown in Fig. 6.4. The deflection criteria and the plots for various sizes of reinforcement are shown in their usual colours. The relative displacements are shown with long and short dashes.

Fig. 6.4: Relative displacements of Vulcan models – 9m x 6m Slab Panel (R60)
From the figure, the A142 mesh is inadequate for 60 minutes when evaluated against any of the deflection criteria. The A193, A252 and A393 meshes satisfy the TSLAB deflection criterion while the A252 mesh just satisfies the span/20 criterion.

Fig. 6.5 shows a comparison of the relative displacements with the Bailey-BRE models. The comparison confirms the earlier observation; for lower reinforcement mesh sizes, the Vulcan models perform better than the Bailey-BRE models, while the reverse occurs with higher reinforcement ratios. It is also observed that a very close comparison is obtained between the two models with the A252 mesh, which coincidentally is the minimum reinforcement mesh size to satisfy the span/20 deflection criterion.

![Relative displacements with Bailey-BRE models](image)

**Fig. 6.5: Relative displacements with Bailey-BRE models – 9m x 6m Slab Panel (R60)**

**9m x 9m Slab Panel**

The vertical deflections required by the Bailey method are plotted in Fig. 6.6. For this slab panel, the required minimum reinforcement mesh size was A193 (Newman et al., 2006). It is observed that, with a decrease in the aspect ratio as compared to the previous slab panels, the general performance of the slab panel is improved in the Bailey-BRE model. However, the specified A193 minimum reinforcement fails according to the TSLAB criterion, at 41 minutes. By the maximum allowable deflection criterion, the slab panel attains 80 minutes. It is also noted again that an increase in reinforcement from A193 to A252 increases the slab panel rating from one below 60 minutes to one over 90 minutes.
The comparison of the *Vulcan* and Bailey 9m x 9m models is shown in Fig. 6.7. Closer comparisons are observed between the two methods for reinforcement sizes below A252, within 25 to 40 minutes. The *Vulcan* 9m x 9m model appears to develop early failure as compared to the 9m x 6m model. This is because there is an increase in the loaded area in the 9m x 9m model. This phenomenon is not observed the BRE model, as it does not monitor the protected perimeter beams. Failure of the secondary beams occurs at 73 minutes with their steel temperature at 621 °C.

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**Fig. 6.6: Bailey-BRE Results – 9m x 9m Slab Panel (R60)**

---

**Fig. 6.7: Comparison of Results and Edge beam failure – 9m x 9m Slab Panel (R60)**
The general trend observed in earlier slab panels is confirmed here. Lower reinforcement mesh sizes show that the Bailey method is conservative, while higher mesh sizes overestimate slab panel capacities in the BRE approach. The *Vulcan* results, however, show that slab panel geometry affects slab capacity more than the reinforcement capacity. The 9m x 9m slab panel (with an aspect ratio of 1.0) emphasises the contributions of reinforcement size to slab panel capacity; a significant increase in capacity is seen from 30 minutes with the A142 mesh to 59 minutes with the A393 mesh when the TSLAB deflection criterion is used.

Failure of the slab panels in the *Vulcan* model is re-examined, using relative displacements of the centre of the slab with respect to the midpoints of the secondary beams (Fig. 6.8). From the figure, it is observed that reinforcement sizes above A142 satisfy the TSLAB and original BRE limits. However, the A193 mesh is not adequate if a span/20 deflection criterion is used.

Relative displacements and the Bailey-BRE required deflections are shown in Fig. 6.9. The results show that for the lighter meshes (A142 and A193) *Vulcan* panels exhibit higher load capacities, while they compare closely with the BRE Method for heavier meshes (A252 and A393). The analyses also show that the BRE method is very sensitive to increases in reinforcement area. A close comparison is obtained with the A252 mesh, while the other mesh sizes seem to be divergent. Again, coincidentally,
the slab panel reinforced with the A252 mesh is the least reinforced panel to exceed the span/20 criterion when relative displacements are evaluated (Fig. 6.9).

![Graph showing relative displacements](image)

**Fig. 6.9: Relative displacements with Bailey-BRE models – 9m x 9m Slab Panel (R60)**

### 9m x 12m Slab Panel

Results for the R60 9m x 12m slab panel are presented in Figs. 6.10 to 6.13. The Level 1 design guide (Newman *et al.*, 2006) specifies a minimum reinforcement mesh of A252. Fig. 6.10 shows that the A252 mesh is inadequate, and only attains 35 minutes in fire. With the dependence of the limiting deflection (Equation 5.1) on the lengths of the spans, TSLAB suggests that having longer beams yields lower slab panel capacities in fire, hence the need for larger meshes.

The comparison of the Bailey-BRE method with *Vulcan* is shown in Fig. 6.11. It is observed that the Bailey-BRE method predicts very high fire resistance times with high reinforcement ratios as compared to *Vulcan*. The results show that, with meshes of A142, A193 and A252, the results of both models are comparable, while with A393 the results are divergent. Again, the *Vulcan* analyses indicate higher capacities with A142, A193 and A252 meshes.
Runaway failure occurs in the *Vulcan* models at 68 minutes, when the protected secondary beams have reached temperatures of 594°C. Similar to the 9m x 6m and 9m x 9m slab panels, the relative displacements of the *Vulcan* models are compared with the deflection criteria and the Bailey-BRE method. The results are shown in Figs. 6.12 and 6.13 respectively.
From Fig. 6.12, the span/20 criterion is only satisfied by the A393 mesh, while A193, A252 and A393 satisfy the TSLAB deflection and the BRE limit. Comparisons of the relative displacements with the Bailey method in Fig. 6.13 show a close correlation between the A393-reinforced Bailey-BRE panel and the equivalent *Vulcan* slab panel. It is noted that relative displacements of the A252 meshes in the 9m x 6m and 9m x 9m slab panels also compared closely with the Bailey-BRE panels. In Fig. 6.12, the relative displacement of the *Vulcan* A393 model passes the span/20 deflection.

**Fig. 6.12: Relative displacements of *Vulcan* models – 9m x 12m Slab Panel (R60)**

**Fig. 6.13: Relative displacements with Bailey-BRE models – 9m x 12m Slab Panel (R60)**
The comparisons above show that the finite element method shows marginal increases in slab panel capacity with increasing reinforcement mesh size. The Bailey-BRE method, on the other hand, shows huge gains in slab panel resistance with higher mesh sizes, even when compared to the relative displacements of the finite element analyses. It is observed that Vulcan shows higher capacities for mesh sizes A142 and A193, but lower capacities with A252 and especially A393 in comparison to the Bailey-BRE method. Relative displacements of the Vulcan and Bailey-BRE models show close comparisons for A252 meshes in the 9m x 6m and 9m x 9m slab panels while similar comparison is seen with the A393 mesh in the 9m x 12m panel. The results show that large increases in slab panel capacity are seen as reinforcement areas increase from 193mm$^2$/m to 393mm$^2$/m. Further analyses are therefore conducted to evaluate why this is the case.

6.3.2 Effects of reinforcement area
As a consequence of the observations made in the previous sections, fictitious reinforcement meshes have been used in addition to the A142, A193, A252 and A393 mesh sizes in order to examine the effects of increasing reinforcement ratios on slab panel resistance. Therefore, the entire range of mesh areas is: 142, 166, 193, 221, 252, 284, 318, 354 and 393 (mm$^2$/m). Their bar diameters are 6, 5.5, 7, 6.5, 8, 8.5, 9, 9.5 and 10 (mm) respectively and these are all spaced at 200mm.

The results in this section therefore show comparisons of failure times of the BRE and Vulcan analyses by finding the equivalent times at which their deflections exceed the deflection criteria of TSLAB, the BRE vertical deflection limit and span/20. The solid lines show results from the membrane action method. Those from Vulcan are shown in dashed lines. The green lines show the failure times as compared to the span/20 criterion; dark blue lines show failure times relative to TSLAB and red lines show failure times using the BRE limit.

Fig. 6.14 shows a comparison of failure times of the Vulcan and Bailey-BRE methods for a 60-minute fire-resistant design of the 9m x 6m slab panel. The plots show the times at which the deflections of the various models exceed the TSLAB, span/20 and maximum allowable BRE limits. From the graph, it is observed that there is limited increase in slab panel resistance with the Vulcan models, while the Bailey models indicate that the slab panel can safely attain at least 133 minutes when a minimum reinforcement of 252mm$^2$/m is used.
A comparison of failure time versus area of reinforcement for the two 9m x 9m slab panel models is shown in Fig. 6.15. Failure times of the slab panel with lower reinforcement areas compare closely for both models, and increase disproportionately in the Bailey model with higher mesh sizes. While the Bailey model can achieve 150 minutes’ fire resistance with an A252 mesh, the Vulcan model only achieves its 60-minute rating with an A393 mesh.
The discrepancy in the two sets of results is due to the shape of the deflections-time plots for the Bailey-BRE model. The steep initial deflections, due to the loss of strength of the unprotected beams, reduces later, with the enhancement of the slab, keeping deflections steady until a time when the reinforcement is hot enough to cause considerable reductions in yield-line capacity. The *Vulcan* model, on the other hand, fails quickly because of the loss of strength of the protected beams on the panel perimeter.

It is observed in comparing failure times for the 9m x 12m slab panel that the close comparison for ‘smaller’ meshes, observed in the 9m x 6m and 9m x 9m slab panels, extends to about 252$\text{mm}^2/\text{m}$ with the TSLAB deflection limit and 318$\text{mm}^2/\text{m}$ with the span/20 criterion (Fig. 6.16). After exceeding these reinforcement sizes the failure times of the two models become divergent.

![Graph showing comparison of failure times](image)

**Fig. 6.16: Comparison of failure times – 9m x 12m slab panel (R60)**

The Bailey method calculates enhancements of the theoretical yield-line load at each time-step. Therefore lower reinforcement sizes require larger enhancements, which in turn require larger deflections. These deflections are all based on the assumption that the edge beams do not deflect. As observed in Chapter 5, this vertical support cannot be maintained indefinitely, as deflections will be generated through the loss of beam stiffness under a constant fire limit state loading. Further investigations of the influence of reinforcement on slab panel resistance are therefore conducted with *Vulcan* models, as they show the true behaviour of slab panel behaviour in fire.
From Figs. 6.14 to 6.16, the higher failure times of the Bailey-BRE method, especially with higher reinforcement ratios, show that larger mesh sizes are necessary. However, these large mesh sizes overestimate the capacity of the slab panel. It has therefore been necessary to investigate the effect of the area of reinforcement on slab panels designed for fire resistances other than 60 minutes. The research, therefore, examined slab panels designed for 90 minutes (R90) and 120 minutes (R120), using design criteria similar to those for 60 minutes. The minimum requirements for each design, and the resulting beam section properties, are shown in Tables 6.3 and 6.4 for the 90-minute design and Tables 6.5 and 6.6 for the 120-minute design.

<table>
<thead>
<tr>
<th>Slab Panel size</th>
<th>9m x 6m</th>
<th>9m x 9m</th>
<th>9m x 12m</th>
</tr>
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<tbody>
<tr>
<td>Dead load (kN/m²)</td>
<td>4.57</td>
<td>4.57</td>
<td>4.57</td>
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<tr>
<td>Live load (kN/m²)</td>
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<tr>
<td>Additional load (kN)</td>
<td>18</td>
<td>41</td>
<td>58</td>
</tr>
<tr>
<td>Beam design factor</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Min. Mesh size</td>
<td>A252</td>
<td>A252</td>
<td>A393</td>
</tr>
</tbody>
</table>

Table 6.3: Slab panel design requirements (R90)

<table>
<thead>
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<th>Slab Panel size</th>
<th>9m x 6m</th>
<th>9m x 9m</th>
<th>9m x 12m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead load (kN/m²)</td>
<td>4.81</td>
<td>4.81</td>
<td>4.81</td>
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<td>Live load (kN/m²)</td>
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<td>5.0</td>
<td>5.0</td>
</tr>
<tr>
<td>Additional load (kN)</td>
<td>20</td>
<td>45</td>
<td>63</td>
</tr>
<tr>
<td>Beam design factor</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Min. Mesh size</td>
<td>A252</td>
<td>A252</td>
<td>A393</td>
</tr>
</tbody>
</table>

Table 6.5: Slab panel design requirements (R120)
Table 6.6: Beam design data for R120 fire resistance

<table>
<thead>
<tr>
<th>Slab Panel</th>
<th>Beam Type</th>
<th>Beam Section</th>
<th>Load ratio</th>
<th>Limiting temperature</th>
<th>R120 Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>9m x 6m</td>
<td>Intermediate</td>
<td>356x171x45</td>
<td>0.467</td>
<td>622°C</td>
<td>1045°C</td>
</tr>
<tr>
<td></td>
<td>Secondary</td>
<td>356x171x57</td>
<td>0.445</td>
<td>629°C</td>
<td>549°C</td>
</tr>
<tr>
<td></td>
<td>Primary</td>
<td>406x178x60</td>
<td>0.453</td>
<td>626°C</td>
<td>550°C</td>
</tr>
<tr>
<td>9m x 9m</td>
<td>Intermediate</td>
<td>356x171x45</td>
<td>0.467</td>
<td>622°C</td>
<td>1045°C</td>
</tr>
<tr>
<td></td>
<td>Secondary</td>
<td>356x171x67</td>
<td>0.452</td>
<td>627°C</td>
<td>550°C</td>
</tr>
<tr>
<td></td>
<td>Primary</td>
<td>533x210x101</td>
<td>0.449</td>
<td>628°C</td>
<td>549°C</td>
</tr>
<tr>
<td>9m x 12m</td>
<td>Intermediate</td>
<td>356x171x45</td>
<td>0.467</td>
<td>622°C</td>
<td>1045°C</td>
</tr>
<tr>
<td></td>
<td>Secondary</td>
<td>457x152x67</td>
<td>0.447</td>
<td>629°C</td>
<td>550°C</td>
</tr>
<tr>
<td></td>
<td>Primary</td>
<td>686x254x170</td>
<td>0.454</td>
<td>626°C</td>
<td>550°C</td>
</tr>
</tbody>
</table>

Table 6.7: Failure times and secondary beam temperatures of slab panels

<table>
<thead>
<tr>
<th>Slab Panel</th>
<th>Failure Time (min)</th>
<th>Intermediate beam temperature (°C)</th>
<th>Secondary beam temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R60 9m x 6m</td>
<td>82</td>
<td>983</td>
<td>663</td>
</tr>
<tr>
<td>9m x 9m</td>
<td>73</td>
<td>963</td>
<td>621</td>
</tr>
<tr>
<td>9m x 12m</td>
<td>68</td>
<td>952</td>
<td>594</td>
</tr>
<tr>
<td>R90 9m x 6m</td>
<td>124</td>
<td>1051</td>
<td>673</td>
</tr>
<tr>
<td>9m x 9m</td>
<td>113</td>
<td>1036</td>
<td>637</td>
</tr>
<tr>
<td>9m x 12m</td>
<td>101</td>
<td>1018</td>
<td>593</td>
</tr>
<tr>
<td>R120 9m x 6m</td>
<td>163</td>
<td>1103</td>
<td>673</td>
</tr>
<tr>
<td>9m x 9m</td>
<td>148</td>
<td>1083</td>
<td>634</td>
</tr>
<tr>
<td>9m x 12m</td>
<td>136</td>
<td>1067</td>
<td>601</td>
</tr>
</tbody>
</table>

Fig. 6.17 shows a comparison of the times at which the central deflections of the 9m x 6m *Vulcan* slab panel exceed the deflection limits of TSLAB (shown in dark blue), the
BRE maximum allowable deflection (in red) and the span/20 deflection criterion (in green), relative to the failure of the supporting protected secondary beams. The comparisons are shown for 60 minutes, 90 minutes and 120 minutes. These are shown by the chain-dot lines dashes for 60 minutes and then broken lines and solid lines for 90 minutes and 120 minutes respectively.

![Graph](image)

**Fig. 6.17: Vulcan failure times relative to area of reinforcement (9m x 6m slab panel)**

It is shown in Fig. 6.17 that very little difference is observed when reinforcement sizes are increased beyond 220mm²/m for the R60 case. For R90 and R120 cases, the lower mesh sizes do not contribute much to slab panel resistance, and failure is likely due to the initial thermal deflections exceeding the deflection limits. Mesh sizes above 260mm²/m show significant improvements in slab panel capacity for those cases. For example, the A393 mesh will fail at 70 minutes and 90 minutes for a 90- and 120-minute fire-resistant design respectively. The general lower slab panel capacities are because of the lower limiting values of the deflection criteria used.

The results of the Vulcan analyse in Fig. 6.17 can be contrasted with those of the Bailey-BRE method in Fig. 6.18. It is observed that comparable results are obtained when reinforcement area is below 200mm²/m. Depending on the fire resistance period, increasing reinforcement area above 200mm²/m generates huge capacities in the Bailey-BRE method.
Fig. 6.18: Bailey-BRE failure times relative to area of reinforcement (9m x 6m slab panel)

Fig. 6.19 shows the comparisons of failure times for the 9m x 9m slab panel. There is a larger variation, as compared to those of the 9m x 6m slab panel. Considerable increases in capacity are observed with mesh sizes above 220mm$^2$/m. However, this effect disappears after about 280mm$^2$/m. The results indicate that smaller meshes should be used for lower fire resistances, and larger meshes for high ratings. The results also show that increasing mesh size in the range 280mm$^2$/m – 393mm$^2$/m does not significantly influence failure times.

Fig. 6.19: Vulcan failure times relative to area of reinforcement (9m x 9m slab panel)
Chapter 6: Effects of reinforcement ratios

Fig. 6.20: Bailey-BRE failure times relative to area of reinforcement (9m x 9m slab panel)

A comparison of the failure times of the Bailey-BRE model is presented in Fig. 6.20. Deductions similar to those of the 9m x 6m slab panel can be made.

Fig. 6.21: Vulcan failure times relative to area of reinforcement (9m x 12m slab panel)

Failure times, using the various deflection criteria relative to slab panel edge beam failure in the 9m x 12m slab panel are presented in Fig. 6.21. The 9m x 12m slab panel shows steady increase in slab panel capacity when reinforcement area is
increased in slab panels designed for 60 minutes’ fire resistance. For 90 and 120 minutes, the lower reinforcement mesh sizes show early failure. A sharp increase in slab panel capacity is seen with reinforcement areas above 240mm$^2$/m. This increase in slab panel resistance continues until about 350mm$^2$/m, above which any increase in mesh area makes very little contribution to slab panel resistance. Because of the geometry of this relatively larger panel, the span/20 criterion seems to produce very low failure times. The improved survival times for mesh above 240mm$^2$/m suggests that the minimum reinforcement area is proportional to the dimensions of the slab panel.

In comparing the results of Bailey-BRE predictions of the 9m x 6m (Fig. 6.18), 9m x 9m (Fig. 6.20) and 9m x 12m (Fig. 6.22) slab panels, it is observed that for the larger panel, the predictions of the *Vulcan* and Bailey-BRE models are close for mesh sizes below 250mm$^2$/m. However, beyond this mesh size huge capacities are again obtained in the simple model.

![Fig. 6.22: Bailey-BRE failure times relative to area of reinforcement (9m x 12m slab panel)](image)

In general, structural elements lose strength and stiffness when exposed to high temperatures. The level at which reinforcement is placed in a composite slab cross-section can aid its capacity or work to its detriment, depending on the thermal exposure of the reinforcement. In general, due to the influence of the reinforcement towards failure, the reinforcement is normally placed 15mm to 40mm above the top flange of
the steel deck to keep it fairly well insulated from heat during fires (Newman et al., 2006).

However, the higher the position of the reinforcement, the lower is the slab panel capacity at ambient temperature. Although the mesh is usually only used for crack control in curing, the larger lever arm obtained from low placement of reinforcement may be compensated by higher temperatures. Fig. 6.24 therefore shows a comparison of the effects of increasing reinforcement area on slab panel capacities by examining the effect of higher temperatures. In Fig. 6.23, the 9m x 9m slab panel designed for 90 minute fire resistance is re-analysed with the reinforcement lowered by 25mm, as shown in Fig. 6.23.

Fig. 6.23: Slab cross-section showing lowered reinforcing mesh (R90 design)

Fig. 6.24: Comparison of failure times and reinforcement temperature
In this figure, the normalised failure times of the 9m x 9m slab panel with average reinforcement depth at 45mm are shown in dashed lines while those of the panel with reinforcement at 70mm depth are shown as solid lines. As usual, the colours show the times at which the *Vulcan* slab panel deflections exceed the limiting deflections of the BRE Limit (in Red), TSLAB (in dark Blue) and span/20 (in Green). Fig. 6.24 shows that for reinforcement mesh sizes between 142mm\(^2\)/m and 200mm\(^2\)/m, higher resistance is obtained by placing the mesh at lower depths. Between 200mm\(^2\)/m and 300mm\(^2\)/m, the mesh at a higher depth outperforms that at the lower depth but the higher lever-arm advantage is restored with mesh sizes above 300mm\(^2\)/m. It should be noted that the highest reinforcement temperature, recorded at the time of failure of the protected secondary beams, was 446°C. This represents an average temperature of the bars in the two directions.

The initial high performance of the panel with reinforcement at lower depth is attributable to the higher bending resistance against the initial thermal bowing, due to the larger lever arm. This phenomenon also accounts for the higher resistance towards failure, caused by the loss of tensile membrane action due to the failure of the protected secondary beams. In-between these two extremes, tensile membrane action dominates the behaviour of the slab panel, and it is observed from Fig. 6.24 that comparable failure times are obtained.

### 6.3.3 Discussions

Generally it is expected that, as aspect ratios approach 1.0, the BRE method should produce conservative solutions, but this does not seem to hold whenever very large spans are concerned or when primary beams exceed the secondary beams in length. The observed ‘jump’ in failure times as heavier reinforcement is used has been investigated further, with a wider variety of reinforcement sizes. Because of the nature of the calculation of the minimum deflection required to achieve an enhanced slab panel capacity, the Bailey method relies heavily on the theoretical yield-line capacity, and therefore limits the deflections beyond which the minimum yield-line capacity is available. It should be noted also that the use of average reinforcement temperatures does not allow higher deflections in the Bailey-BRE method.

The results show that the Bailey method is conservative for the panel sizes studied where A142 and A193 mesh sizes are concerned. This conservatism extends to A252 when larger slab panels are used. The conservatism is lost when larger reinforcement sizes are sued. Comparisons with relative displacements show closer correlation
between the simplified method and the advanced analysis, and holds in practice only if larger deflections of the edge beams are not encountered. In the series of studies shown in the preceding sections, the use of the span/20 deflection criterion has been found to be onerous. The results show that this limit requires the adoption of very high reinforcement ratios and heavily protected slab panel boundaries. The span/20 criterion considers the shorter span of the slab. Protected secondary beams, which have been found to be critical to the failure of slab panels, may not necessarily be oriented in the shorter direction. Therefore, the use of this limit severely limits the allowable design deflection.

In examining the effects of reinforcement ratios on slab panel capacities in fire, it has been observed that for 'small' slab panels (9m x 6m and 9m x 9m), negligible increases in slab panel capacities are obtained when larger reinforcement mesh sizes are used on panels designed for 60-minute fire resistance. However, mesh sizes of at least 220mm²/m are required when these panels are designed for 90 or 120 minutes. As slab panel sizes increase, the results show that larger mesh sizes are required, even when lower fire resistances (R60) are required. The observation therefore suggests that the minimum mesh size required is proportional to the dimension of the slab panel. Section 6.4 therefore examines this theory.

6.4 Relationship between reinforcement sizes and slab spans
To investigate the relationship between reinforcement ratios and slab panel dimensions an analogy is made with cables. It is easily shown that the tensile forces induced in cables of different spans, hanging in catenary between supports on a common level, are directly proportional to the cable spans if the span-deflection ratio is kept constant and the same load intensity is applied in all cases. This is illustrated in the derivations below.
Chapter 6: Effects of reinforcement ratios

Fig. 6.25: Catenary cable analogy

Taking moments about the left support
\[ w'\gamma\left(\frac{L}{2}\right)\left(\frac{L}{4}\right) - T\delta = 0 \]
\[ T = \frac{w'\gamma L^2}{8\delta} \]

but \( T = A f_y \), where \( A = \text{area} \) and \( f_y = \text{Reinf. cement yield strength} \)

\[ \Rightarrow A = \frac{w'\gamma L^2}{8\delta f_y} \]

If \( \Delta = \frac{\text{span}}{\text{vertical deflection}} = \frac{L}{\delta} \)

\[ \Rightarrow A = \frac{w'\gamma L\Delta}{8 f_y} \]

\[ \Rightarrow A \propto L \]

The above relationship indicates that, for the same grade of reinforcing steel, mesh cross-sectional areas should also increase in proportion to the spans as the sizes of slabs increase, and that the reinforcement ratio as a proportion of the concrete area is almost irrelevant. To investigate this, three square slab panels, reinforced with varying reinforcement areas, are analysed to evaluate the extents of central tensile areas that exist within them at span/deflection ratios of 30, 20 and 10.

6.4.1 Analyses
The three slab panels considered have dimensions 6m x 6m, 9m x 9m and 12m x 12m. The panels were designed such that their edge beam temperatures reached a maximum of 550°C at 60 minutes. For a fire limit state load of 5.83kN/m², the load
ratios of the protected edge beams were maintained between 0.40 and 0.45. Unprotected intermediate beams had ambient-temperature utilisation values between 0.80 and 0.85. Following the average depth approach, the concrete slab profile of Fig. 5.2 was used with the corresponding slab and reinforcement temperatures shown in Fig. 5.5. Reinforcement areas ranged from 142mm$^2$/m to 393mm$^2$/m (in both directions). The corner support condition used in earlier analyses was employed in the study here. A sketch of the 9m x 9m slab panel with centrelines perpendicular and parallel to the intermediate unprotected beams is shown in Fig. 6.26. Due to the symmetric nature of the structural problem, quarter section of the panels were analysed with the necessary symmetric boundary conditions along the centrelines X-X and Y-Y.

Membrane tractions (forces per unit lengths) across the Y-Y centrelines of the slab panels were examined, to exclude the effects of axial forces in the unprotected beams. In the 9-noded Vulcan slab elements there are 9 Gaussian integration points at each layer level. Stresses in the slab element are calculated at these points, and integrated through the depth of the slab to determine the force per unit length at a particular gauss point. Along the length of the isoparametric slab element, the Gaussian integration points are at the proportions $-\sqrt{3/5}$, 0 and $+\sqrt{3/5}$. Since one quarter of each slab panel is analysed, the membrane tractions at the Gauss points nearest to the Y-Y centreline are examined to find the length of the central tensile area.
At each Gauss point principal values of membrane tractions are generated. These principal values lie perpendicular to each other with the maximum principal traction oriented at an angle to the X–direction. In the present study, the membrane traction in the X–direction, across the Y-Y centreline, is sought to isolate the effects of axial forces in the unprotected composite beams which lie in the Y–direction (see Fig. 6.26). The three results across one element are not necessarily continuous with those preceding or succeeding it, due to the nature of the nonlinear finite element analysis. A polynomial of best fit is therefore plotted over the membrane traction values to determine the length of the tensile area. Fig. 6.27 shows the plot used to extract the length of the tensile area in the 9m x 9m slab panel reinforced with an A142 mesh.

![Graph showing membrane tractions across Y-Y centreline of A142 reinforced 9m x 9m panel](image)

**Fig. 6.27: Membrane Tractions across Y-Y centreline of A142 reinforced 9m x 9m panel**

In Fig. 6.27, the results at the Gauss points are shown as discrete points while the polynomial of best fit is shown as a continuous curve. The results are obtained for span/deflection ratios of 30, 20 and 10. These are shown in dark blue, red and green respectively. As expected, the central area of the slab experiences tension with a surrounding ring of compression. The tensile area spreads as vertical deflections increase, resulting in increased compressive membrane forces per unit length towards the edge of the slab panel. For a span/deflection ratio of 30, the central tensile area spans 3400mm (2 x 1700mm). This increases to 4000mm with a span/deflection ratio of 20 and extends to about 6560mm when the central deflection reaches 900mm.
6.4.2 Results

Following the derivation at the beginning of this section, and considering that quarter slab sections are used for the analyses, half-lengths of the induced tensile areas are sought. Therefore the half-lengths of the induced tensile area can be obtained as:

\[
\frac{L}{2} = \frac{4Af_\gamma}{w\gamma\Delta}
\]  

(6.4)

The theoretical tensile widths are plotted in Fig. 6.28. The dark blue, red and green lines show the half-widths of tensile areas that can be supported at span/deflection ratios of 30, 20 and 10, with reinforcement areas ranging from 142mm$^2$/m to 393mm$^2$/m.

![Graph showing theoretical prediction of the relationship between reinforcement area and the induced central tensile area](image)

**Fig. 6.28: Theoretical prediction of the relationship between reinforcement area and the induced central tensile area**

It is observed that a tensile area with a radius of over 11m could be supported with sufficient vertical deflection and an A393 reinforcement mesh.

Fig. 6.29 shows a plot of the theoretical tensile area radii at a span/deflection ratio of 30 and the results obtained with the three *Vulcan* slab panels. Results for the 6m x 6m slab panel are shown as filled square boxes, while those for the 9m x 9m panel are shown as triangles, and the circular dots show the radii for the 12m x 12m panel. It is observed that an increase in mesh size for the 6m x 6m slab panel initially results in proportional increase of the tensile radius up to a reinforcement size of 193mm$^2$/m. Proportional increases in the central tensile area of the 9m x 9m slab panel are also observed. This proportionality is lost with reinforcement meshes above 300mm$^2$/m. From the figure, the radii of tensile area induced with smaller reinforcement sizes, in
the 12m x 12m slab panel, are more than their theoretical values. With higher reinforcement, however, more accurate predictions of the central tensile area are given by the established relationship.

![Graph showing comparisons of induced tensile area at span/deflection of 30](image1)

**Fig. 6.29: Comparisons of induced tensile area at span/deflection of 30**

A similar trend is observed when the span/deflection ratio is 20 (Fig. 6.30). The closest comparison to the theoretical prediction is obtained with the 12m x 12m slab panel, while the worst occurs with the 6m x 6m panel. A linear correlation is observed for the results of the 9m x 9m panel, although these values are below the theoretical prediction.

![Graph showing comparisons of induced tensile area at span/deflection of 20](image2)

**Fig. 6.30: Comparisons of induced tensile area at span/deflection of 20**
Fig. 6.31 shows a similar comparison for span-deflection limit of 10 of the theoretical prediction of the relationship between reinforcement size and the induced central tensile area, and the results obtained with the three *Vulcan* slab panels of 6m x 6m, 9m x 9m and 12m x 12m.

![Graph showing comparisons of induced tensile area at span/deflection of 10](image)

**Fig. 6.31: Comparisons of induced tensile area at span/deflection of 10**

Apart from a modest increase in the length of the tensile area with mesh sizes up to 193mm$^2$/m with the *Vulcan* 12m x 12m slab panel model, the results show poor comparisons with the theoretical model. At span/deflection ratio of 10, the slab panels are supposed to attain deflections of 600mm, 900mm and 1200mm respectively. However, with corner supports and protected boundaries, vertical support of the protected secondary beams is lost at midspan deflections ranging from 399mm to 434mm for the 6m x 6m slab panel, 599mm to 738mm for the 9m x 9m slab panel and 819mm to 1034mm for the 12m x 12m slab panel, depending on the area of reinforcement. Thus, the values shown in Fig. 6.31 were recorded during failure of the slab panel, and hence the deviation from the theoretical prediction.

The effect of reinforcement ratios on vertically supported slab panels is examined in Fig. 6.32. In the figure, comparisons of the various edge-supported panels and the theoretical prediction for a span/deflection ratio of 30 are made. The theoretical prediction is shown as the light blue line with the dark blue curve showing the variation of the induced central tensile area with increasing reinforcement size for the 6m x 6m slab panel. Results for the 9m x 9m and 12m x 12m slab panels are shown in red and green respectively. Comparisons are not made for either the 20 or 10 span/deflection...
ratios because full sets of results could not be obtained due to the edge beam failure of the slab panels before the attainment of the required high deflections.

![Graph](image)

**Fig. 6.32: Comparisons of induced tensile area at span/deflection of 30 with continuous vertical support along the slab panel boundaries**

It is observed that the largest gain in the central tensile area is seen in the 12m x 12m slab panel with the radius of its tensile region increasing from 2000mm with an A142 mesh to 6000mm with an A393 mesh. With the 6m x 6m slab panel, little benefit is obtained by the increased reinforcement area. It is also observed that reinforcement mesh areas above 320mm²/m do not influence the central tensile area. With the 9m x 9m slab panel, a sharp increase is seen up to 252mm²/m, followed by a reduction in capacity and then a modest increase in tensile capacity of the slab panel within a large range of reinforcement areas.

The analyses in this section have so far shown that there is a limit to the enhancement provided by increasing the reinforcement ratios of slab panels. The analyses show that increasing slab panel dimensions with corresponding increases in reinforcement area, can generate considerable capacity.

### 6.5 Summary

This chapter has investigated the effects of reinforcement ratios on slab panel capacities in fire. The chapter has examined the recent improvements in the Bailey method in comparison to *Vulcan*, the University of Sheffield’s specialist fire engineering software. The comparison shows that, for smaller reinforcement mesh sizes, the
Bayley method is conservative, but that it gives very optimistic slab panel capacities with high reinforcement ratios. *Vulcan*, on the other hand, shows modest increases in capacity with increasing reinforcement size, even when the supporting edge beams are assumed to remain intact throughout fire exposure.

Further investigations carried out with a larger variation of mesh sizes has confirmed the disproportionate increase in slab panel capacity predicted with larger reinforcement meshes by the Bailey-BRE method, especially for small panels. *Vulcan* investigations have further revealed that higher reinforcement mesh sizes are required for large panels, and when higher fire resistances times are needed on small-sized panels. An investigation into the relationship between reinforcement mesh sizes and slab dimensions has also shown that higher tensile tractions increase with increasing reinforcement and larger slab spans.

The chapter has therefore found that, even with the recent advances of the Bailey-BRE method, it loses conservatism where higher reinforcement ratios are concerned. It has also been found that larger panels require higher reinforcement areas to generate sufficient tensile capacity, while higher reinforcement mesh sizes are only required by small panels when higher fire resistances are required.
Chapter 7: Conclusions and Recommendations

7 Conclusions and Recommendations

Developments in structural fire engineering towards more performance-based design methods have identified tensile membrane action of composite slabs at elevated temperatures as a primary load-bearing mechanism, if the slab is adequately supported vertically, and undergoes biaxial bending. The use of this mechanism in the design of composite steel-framed buildings yields benefits in economy while ensuring that minimum safety requirements are met. A number of design methods have been developed, which incorporate tensile membrane action at elevated temperatures. Prominent among these is the Bailey-BRE method, which has been adopted by the Steel Construction Institute (SCI). Depending on the properties of a particular slab panel, the method may either be conservative or unconservative. The demerits of the approach have been attributed to a lack of adequate consideration of the development of tensile membrane action from a thermal perspective rather than as an extension to the ambient-temperature method.

The series of studies in this thesis were designed to highlight the development of tensile membrane action from a mainly thermal perspective. The conclusions from the various studies are mentioned in subsequent sections, and recommendations on future research are made at the end of the chapter.

7.1 Conclusions

7.1.1 Experimental studies on thin slabs in fire

Experiments were conducted on model-scale slabs of nominal dimensions 900mm x 600mm x 15mm, to assess the influence of thermal gradients acting alone on the development of tensile membrane action. Loaded tests were also carried out at elevated temperatures with smooth and deformed reinforcing wires at both low and high reinforcement ratios to assess their contributions to thin-slab behaviour in fire. The following conclusions were drawn:

- A thermal gradient, acting alone can induce a considerable amount of tensile membrane action in concrete slabs at elevated temperatures. The gradient also induces transverse tension cracking across the short span of the slab.
- Lower bond strength between reinforcement and concrete, and higher reinforcement ductility, produce slabs with higher capacities than those with higher bond strength or low ductility.
Chapter 7: Conclusions and Recommendations

• Increasing reinforcement ratios increase concrete slab capacities at elevated temperature. For relatively low ratios (up to about 0.2%), proportional increases are observed when the ratios increase. Above 0.2%, the increase in capacity reduces with further increase in reinforcement ratio.

• Conservative estimates of test results were obtained with the Bailey-BRE method when low reinforcement ratios were used. Higher reinforcement ratios generated unconservative predictions. Vulcan, however, gave good approximations irrespective of the reinforcement ratio.

• Two failure modes were observed. Lightly reinforced slabs failed by fracture of reinforcement in the centre of the slab across the shorter span, while highly reinforced slabs failed by corner crushing of concrete.

• The compressive failure mechanism was only observed at ambient temperature. At elevated temperature, highly reinforced slabs only showed a distribution of discrete cracks oriented in yield line patterns with no clear transverse tension cracking.

7.1.2 Thermal gradients
The possibility of quantifying tensile membrane action in horizontally-unrestrained simply-supported slabs by exposure to thermal action alone was explored with a simple classical method of calculating the capacity of composite slabs in fire. The method, developed with the Rayleigh-Ritz approach, established the following:

• Differential thermal expansions through the depth of a slab induce tensile membrane action.

• The effects of this thermal action on displacements, membrane tractions and stresses through the depth of the slab can be quantified.

• Horizontal restraints are not required for tensile membrane action, as the compressive ring develops irrespective of the presence of axial restraint along the slab’s boundary.

• Edge beams and their connections are, therefore, not required to provide lateral restraint to composite slabs.

• Good predictions of slab response to thermal actions were obtained with the analytical approach, especially in central areas of the slab.

• The method is seen as a prelude to a composite design method that accounts for both thermal and mechanical actions at elevated temperatures.
Towards the edges of the slab, the method was found to be lacking. Large numbers of shape functions were required to adequately represent stresses in the region of the compressive ring. As the numerical process has proved to be difficult to implement, the current state of this method needs considerable computational effort to establish accurate solutions.

### 7.1.3 Slab stress patterns at elevated temperatures

As the Rayleigh-Ritz method was very time consuming, *Vulcan* was used to extend the study on the effects of thermal gradients. Two through-depth thermal distributions were considered together with different slab axial stiffnesses and seven thermal gradients (1 °C/mm – 7 °C/mm). The conclusions were:

- Thermal gradients induce compressive stresses in the lower layers of slabs with corresponding tensile stresses in the upper layers.
- In general, increasing the thermal gradient increases the magnitudes of the stresses induced in the slab, thereby generating larger deflections and membrane tractions which help sustain tensile membrane action.
- When axial restraints are present, the induced stresses are counteracted by the restraint to mean thermal expansion.
- Restraints to thermal expansion therefore dominate the behaviour of the slab until significant vertical deflections occur, releasing the lateral compressive force (which depends on the amount of axial restraint present), after which the thermal gradients dominate the behaviour of the slabs once more.

### 7.1.4 Slab panel vertical support

The Bailey-BRE method assumes that edges of slab panels remain horizontal throughout fire exposure. However, in practice these edges consist of protected beams, which lose strength and stiffness in fire. The investigation therefore explored the provision of adequate vertical support along slab panel boundaries and develops an edge-beam failure mechanism for composite slab panels in fire. The major conclusions were:

- Tensile membrane action is lost when slab panel edge beams experience significant vertical deflections.
- Considerable restraint is provided either by vertically supported edges or continuous slab panels.
• Slab panels at the corner, or along the edge, of a building are more susceptible to edge-beam failures than those in the interior of the building.
• It has been found that a combination of imposed loads and material degradation cause failure.
• The specification of a protected beam temperature of 620°C at the required fire resistance time is not necessarily sufficient to ensure fire resistance.
• Restraint from adjacent slabs is beneficial for interior slab panels, but corner or edge panels may require increased protection material thicknesses.
• A plastic failure mechanism for slab panel edge beams has therefore been proposed.
• For the collapse mechanism, failure of the parallel arrangement of secondary beams precedes failure of the primary beams.
• A discussion of more complex folding mechanisms is also given.

7.1.5 Effects of reinforcement ratios

The research presents a review of the improvements to the Bailey-BRE method in terms of the effects of increasing reinforcement ratios on slab panel capacities in fire. Three slab panel aspect ratios were examined in relation different fire resistant times, the deflection limits of TSLAB, the BRE allowable deflection and the span/20 criterion and various reinforcement mesh sizes. Comparisons were made with Vulcan. An investigation into the relationship between reinforcement mesh sizes and slab panel size was conducted. The major conclusions were:
• For smaller reinforcement mesh sizes, the Bailey-BRE method is found to be conservative. However, it is found to give very optimistic capacities with high reinforcement ratios.
• *Vulcan* shows modest increases in capacity with increasing reinforcement size.
• A disproportionate increase in slab capacity is observed with the Bailey-BRE method when large mesh sizes are used, especially for small-sized panels.
• The position of the reinforcement does not make any significant contribution to the capacity in tensile membrane action, but may influence slab resistance at ambient-temperature and towards failure (when protected beams lose significant stiffness).
• The central area of tensile tractions is observed to increase with increasing reinforcement size.
• At a particular span/deflection ratio, the radius of the central tensile area increases proportionally with the span of the slab.
• The Bailey-BRE method is found to lose its conservatism when higher reinforcement ratios are considered.
• Larger reinforcement sizes are found to be essential to larger slab panels. For small-sized panels, larger meshes are only beneficial where higher fire resistances are required.

7.2 Recommendations

7.2.1 Bond-slip between concrete and reinforcement
The experimental program showed that weakly-bonded reinforcement helps to generate higher slab capacities in fire. A large-deflection slab model which considers reinforcement bond-slip is required in finite element codes such as *Vulcan*.

7.2.2 Extension of the analytical Rayleigh-Ritz model
The analytical study on thermal gradients established that the contribution of thermal actions to tensile membrane action can be quantified. A numerical procedure is possible if the complex equations are simplified or decoupled. This could therefore provide a simple model which can effectively consider both thermal and mechanical effects in the prediction of slab behaviour at elevated temperature.

7.2.3 Extension of the edge beam plastic failure mechanism
The present mechanism considers only beams in its prediction and ignores any contribution from the reinforcement. An extension of the folding prediction to include the reinforcement and all the other possible folding mechanisms will offer a better prediction of slab panel folding failure, which could serve as a design check within the Bailey-BRE method.
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